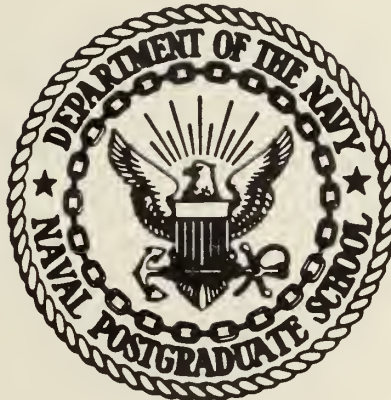


United States Naval Postgraduate School



AERODYNAMIC DESIGN OF SYMMETRICAL
BLADING FOR THREE-STAGE AXIAL FLOW
COMPRESSOR TEST RIG

by

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ABSTRACT:

This report deals with the aerodynamic design of an axial compressor stage with symmetrical bladings for a research program to investigate tip clearance effects in the three-stage compressor of the Turbo-Propulsion Laboratory, NPS. It establishes the blading data and the stage performance with an iterative three-dimensional approach, and gives design criteria for the drive and the flow measuring device of the test unit. The calculated distributions of the flow properties in the stage will be used for future comparisons with test data.

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1. INTRODUCTION

The three-stage axial flow compressor test rig of the Turbo-Propulsion Laboratory, Department of Aeronautics, NPS, will be used to carry out a Navy sponsored research program on three-dimensional flow phenomena in compressor bladings. This compressor with a tip diameter of 36 in. and a hub/tip ratio of 0.6 is presently equipped with lightly loaded bladings which are of the free-vortex type. A 50 HP two-speed electric motor is now directly coupled to the compressor. Further details of the test rig are given in Ref. 1 (See Bibliography at end of text).

During the first phase of the research program, so-called symmetrical bladings with high aerodynamic loading will be tested. Further, in order to increase the pressure rise in the compressor, primarily to improve the measuring accuracy, the present motor will be replaced by a 150 HP motor that was obtained from surplus. This motor has the following characteristics:

Make: General Dynamics/Electro Dynamics, Bayonne, N. J.

Frame: 505 Y - Type TN

Serial No.: 70105116 A6

Cont. Load: 150 HP, 1180 rpm, 440V/3 phase, 190 Amps

It is planned also to make tests with one, two, or three stages, preferable in such a manner that the full power of the motor can be utilized in all three cases, requiring operations at different speeds. It is intended to make this possible by arranging a V-belt drive with different replaceable pulleys.

The objective of this study is to establish the aerodynamic design of the new blading, and to determine the maximum speeds at which these stages can operate at a driving power of 150 HP. These data will then be used in a future report to design the V-belt drive and the nozzle for the measuring of the compressor flow rate, since Ref. 1 shows that the presently used measuring method gives doubtful results.

The blade loadings will be such that the NASA diffusion factor does not exceed a value of 0.5 for the rotor, or 0.6 for the stator. All three stages will be identical. The flow conditions ahead of and after the blade rows will be determined with the methods of Ref. 2.

2. FIRST APPROXIMATION OF THREE-DIMENSIONAL FLOW PATTERN.

For stages with equal energy input from hub to tip without imposed radial energy gradients, the change of the axial velocity component in radial direction can be obtained from Eq. 16(50) of Ref. 2. For a first approximation it is assumed that the axial components V_{m1} and V_{m2} ahead of and after the rotor blades are equal at the mean radius R_m , and that the radial shift of the stream surfaces can be ignored. The resulting velocity diagram of the blading at the arithmetic mean radius R_m is shown in Fig. 1. Figure 2 represents the conditions for a cylindrical stream surface at an arbitrary radius R between hub and tip. The actual velocity diagram for a stream surface that has the radii R_1 and R_2 ahead of and after the rotor, respectively, is given by Fig. 3. The symbols used in these figures, and in the following derivations, are defined in Ref. 2.

From Eq. 16 (42) of Ref. 2 and by Fig. 1

$$K = R \Delta V_u = R_m \Delta V_{um} = R_m V_m (\tan\beta_{1m} - \tan\beta_{2m}) \quad (1)$$

Also

$$\omega R_m = W_{u1m} + V_{u1m} = W_{u1m} + W_{u2m} = V_m (\tan\beta_{1m} + \tan\beta_{2m})$$

and

$$V_m = \frac{\omega R_m}{\tan\beta_{1m} + \tan\beta_{2m}} \quad (2)$$

thus

$$K = \omega R_m^2 X \quad (3)$$

where

$$X = \frac{\tan\beta_{1m} - \tan\beta_{2m}}{\tan\beta_{1m} + \tan\beta_{2m}} \quad (4)$$

From Eq. 16 (52) of Ref. 2

$$\frac{V_{u1}}{\omega R_m} = -\frac{R_m}{R} \frac{X}{2} + \frac{1}{2} \frac{R}{R_m} \quad (5)$$

$$\frac{V_{u2}}{\omega R_m} = +\frac{R_m}{R} \frac{X}{2} + \frac{1}{2} \frac{R}{R_m} \quad (6)$$

From Eq. 16 (53) of Ref. 2

$$A = -\omega R_m \frac{X}{2}; \quad B = 0; \quad C = \frac{\omega R_m}{2} \quad (7)$$

Introduced into Eq. 16 (50) of Ref. 2 for

$$V_{m1} = V_{m2} = V_m,$$

$$V_{a1}^2 = V_m^2 - 4AC \ln\left(\frac{R}{R_m}\right) - 2C^2 \ln\left[\left(\frac{R}{R_m}\right)^2 - 1\right] \quad (8)$$

and

$$V_{a2}^2 = V_m^2 - 4\left(A + \frac{K}{R_m}\right) C \ln\left(\frac{R}{R_m}\right) - 2C^2 \left[\left(\frac{R}{R_m}\right)^2 - 1\right] \quad (9)$$

From Eqs. 8, 7 and 2

$$V_{a1}^2 = \frac{\omega^2 R_m^2}{(\tan\beta_{1m} + \tan\beta_{2m})^2} + \omega^2 R_m^2 X \ln\left(\frac{R}{R_m}\right) - \frac{\omega^2 R_m^2}{2} \left[\left(\frac{R}{R_m}\right)^2 - 1\right]$$

or

$$\left(\frac{V_{a1}}{\omega R_m}\right)^2 = Y + X \ln\left(\frac{R}{R_m}\right) - \frac{1}{2} \left[\left(\frac{R}{R_m}\right)^2 - 1\right] \quad (10)$$

where

$$Y = \frac{1}{(\tan\beta_{1m} + \tan\beta_{2m})^2} \quad (11)$$

From Eqs. 9, 7, 2, and 10

$$\left(\frac{V_{a2}}{\omega R_m}\right)^2 = Y - X \ln\left(\frac{R}{R_m}\right) - \frac{1}{2} \left[\left(\frac{R}{R_m}\right)^2 - 1\right] \quad (12)$$

Thus, the axial velocity components V_{a1} and V_{a2} at different radius ratios R/R_m can be determined with Eqs. 11 and 12 if the angles β_{1m} and β_{2m} are specified at the mean radius R_m .

From Fig. 2

$$\tan \beta_1 = \frac{\omega R - V_{u1}}{V_{a1}} = \frac{R/R_m - V_{u1}/\omega R_m}{V_{a1}/\omega R_m}$$

With Eq. 5

$$\tan \beta_1 = \frac{\frac{R}{R_m} + \frac{R_m}{R} \frac{X}{2} - \frac{R}{2R_m}}{V_{a1}/\omega R_m} = \frac{\frac{R}{R_m} + \frac{R_m}{R} X}{2(V_{a1}/\omega R_m)} \quad (13)$$

Similarly, with Eq. 6

$$\tan \beta_2 = \frac{\omega R - V_{u2}}{V_{a2}} = \frac{\frac{R}{R_m} - \frac{R_m}{R} X}{2(V_{a2}/\omega R_m)} \quad (14)$$

Also from Fig. 2 and Eq. 5

$$\tan \alpha_1 = \frac{V_{u1}}{V_{a1}} = \frac{\frac{R}{R_m} - \frac{R_m}{R} X}{2(V_{a1}/\omega R_m)} = \tan \beta_2 \frac{(V_{a2}/\omega R_m)}{(V_{a1}/\omega R_m)} \quad (15)$$

and with Eq. 6

$$\tan \alpha_2 = \frac{V_{u2}}{V_{a2}} = \frac{\frac{R}{R_m} + \frac{R_m}{R} X}{2(V_{a2}/\omega R_m)} = \tan \beta_1 \frac{(V_{a1}/\omega R_m)}{(V_{a2}/\omega R_m)} \quad (16)$$

With these flow angles the relative and absolute velocities W_1, V_2 , and V_1, V_2 are known as multiples of ωR_m .

3. PERMISSIBLE FLOW DEFLECTIONS

The NASA diffusion factor D_R of the rotor flow is defined by [Ref. 6]

$$D_R = 1 - \frac{W_2}{W_1} + \frac{\Delta W_u}{2\sigma_R W_1} \quad (17)$$

where σ_R is the rotor solidity. The diffusion factor D_s of the stator flow is

$$D_s = 1 - \frac{V_1}{V_2} + \frac{\Delta V_u}{2\sigma_s V_2} \quad (18)$$

Assuming that the stator solidity σ_s equals σ_R , and that the chord of the stator and rotor blades does not change along the radius, there is

$$\sigma_R = \sigma_s = \sigma_m \frac{R_m}{R} \quad (19)$$

where σ_m is the solidity at R_m .

From Eqs. 1 and 3

$$\Delta V_u = \Delta W_u = \frac{K}{R} = \omega R_m \frac{R_m}{R} X \quad (20)$$

Then from Eq. 17 with 19

$$D_R = 1 - \frac{V_{a2}/\omega R_m}{V_{a1}/\omega R_m} \frac{\cos\beta_1}{\cos\beta_2} + \frac{X \cos\beta_1}{2\sigma_m (V_{a1}/\omega R_m)} \quad (21)$$

Also by Eqs. 18, 19, and 20

$$D_s = 1 - \frac{V_{a1}/\omega R_m}{V_{a2}/\omega R_m} \frac{\cos\alpha_2}{\cos\alpha_1} + \frac{X \cos\alpha_2}{2\sigma_m (V_{a2}/\omega R_m)} \quad (22)$$

For the chosen conditions the diffusion factors D_R and D_s at the mean radius R_m are equal, say, equal to D_m , where by Eqs. 21, 2, and 4

$$D_m = 1 - \frac{\cos\beta_{1m}}{\cos\beta_{2m}} + \frac{(\tan\beta_{1m} - \tan\beta_{2m}) \cos\beta_{1m}}{2\sigma_m} \quad (23)$$

Equation 23 can be rearranged to calculate directly the flow angle β_{2m} for specified values of β_{1m} , D_m , and σ_m . Equation 23 rewritten

$$\frac{1 - D_m}{\cos\beta_{1m}} = \frac{1}{\cos\beta_{2m}} - \frac{\tan\beta_{1m}}{2\sigma_m} + \frac{\tan\beta_{2m}}{2\sigma_m}$$

Hence

$$\tan\beta_{1m} + \frac{2\sigma_m(1 - D_m)}{\cos\beta_{1m}} = \tan\beta_{2m} + \frac{2\sigma_m}{\cos\beta_{2m}}$$

Let

$$E = \tan\beta_{1m} + \frac{2\sigma_m(1 - D_m)}{\cos\beta_{1m}} \quad (24)$$

then

$$E = \frac{\sin\beta_{2m} + 2\sigma_m}{\cos\beta_{2m}} = \frac{\sin\beta_{2m} + 2\sigma_m}{\sqrt{1 - \sin^2\beta_{2m}}}$$

and

$$\sin\beta_{2m} = \frac{-2\sigma_m \quad (+) \quad E \sqrt{1 + E^2 - 4\sigma_m^2}}{1 + E^2} \quad (25)$$

It is possible also to determine the deflection angle

$$\Delta\beta = \beta_{1m} - \beta_{2m} \quad (26)$$

from Eq. 23. This relation rearranged

$$\begin{aligned} 2\sigma_m \cos\beta_{2m}(1 - D_m) &= 2\sigma_m \cos\beta_{1m} - (\tan\beta_{1m} - \tan\beta_{2m})\cos\beta_{1m}\cos\beta_{2m} \\ &= 2\sigma_m \cos\beta_{1m} - (\sin\beta_{1m}\cos\beta_{2m} - \cos\beta_{1m}\sin\beta_{2m}) \\ &= 2\sigma_m \cos\beta_{1m} - \sin(\beta_{1m} - \beta_{2m}) \\ &= 2\sigma_m \cos\beta_{1m} - \sin\Delta\beta \end{aligned}$$

Since from Eq. 26

$$\beta_{2m} = \beta_{1m} - \Delta\beta$$

there is

$$2\sigma_m (1 - D_m) (\cos\beta_{1m} \cos\Delta\beta + \sin\beta_{1m} \sin\Delta\beta) = 2\sigma_m \cos\beta_{1m} - \sin\Delta\beta$$

and

$$\sin\Delta\beta \left[\tan\beta_{1m} + \frac{1}{2\sigma_m (1-D_m) \cos\beta_{1m}} \right] = \frac{1}{1 - D_m} + \cos\Delta\beta$$

with

$$F = \tan\beta_{1m} + \frac{1}{2\sigma_m (1-D_m) \cos\beta_{1m}} \quad (27)$$

there is

$$\sin\Delta\beta = \frac{F (+) \sqrt{(1-D_m)^2 (1+F^2) - 1}}{(1-D_m) (1+F^2)} \quad (28)$$

For chosen values β_{1m} , σ_m , and D_m the flow angles β_{2m} can therefore be determined. The angles β_{1m} and β_{2m} then establish the quantities X and Y of Eqs. 4 and 11, respectively. Introduced into Eqs. 10 and 12 these values can be used to obtain the changes of the axial components V_{a1} and V_{a2} in radial direction. With the angles β_1 and β_2 from Eqs. 13 and 14 it is then possible to calculate the diffusion factors D_R and D_S at particular radii with Eqs. 21 and 22.

4. APPROXIMATE OPERATING CONDITIONS OF COMPRESSOR.

The following relations will be used to establish the main parameters of the blading, which later on will be investigated in more detail.

If it is assumed that V_m is equal to the average through-flow velocity in the compressor annulus, the volume flow rate Q is

$$Q = A_c V_m$$

where A_c is the cross-sectional area of the annulus. With Eq. 2

$$Q = A_c \omega R_m \frac{1}{\tan\beta_{1m} + \tan\beta_{2m}} \quad [\text{ft}^3/\text{sec}] \quad (29)$$

The driving moment M for one stage is

$$\dot{M} = \dot{m} R_m \Delta V_u = \dot{m} K \quad [\text{ft-lb}] \quad (30)$$

Because of the small pressure rise in the compressor the mass flow rate \dot{m} can be taken as

$$\dot{m} = Q \rho_o \quad (31)$$

where ρ_o is the air density at ambient conditions. For z stages ($z = 1, 2, \text{ or } 3$) the necessary horsepower HP is

$$\text{HP} = z \frac{\omega M}{550} = z \frac{\rho_o \omega^3 R_m^3 A_c (\tan \beta_{1m} - \tan \beta_{2m})}{550} \quad (32)$$

At 14.7 psia and 60°F

$$\rho_o = 2.371 (10^{-3}) \quad [\text{slug/ft}^3]$$

With the rotational speed N in rpm

$$\omega = \frac{\pi N}{30}$$

For a tip diameter of 3 feet and a hub/tip ratio of 0.6

$$A_c = \frac{\pi}{4} 3^2 (1 - 0.6^2) = 4.524 \text{ ft}^2$$

$$R_m = 1.5 \frac{(1 + 0.6)}{2} = 1.2 \text{ ft}$$

Then, from Eq. 29

$$Q = \frac{(0.568) N}{\tan \beta_{1m} + \tan \beta_{2m}} \quad [\text{ft}^3/\text{sec}] \quad (33)$$

From Eq. 30

$$\text{HP} = (38.7) z \left(\frac{N}{1000} \right)^3 (\tan \beta_{1m} - \tan \beta_{2m}) \quad (34)$$

Assuming that 135 HP are absorbed at the design point, to have available 150 HP at overload conditions, the design operating speed is obtained from

$$N = \frac{1516.6}{\left[z (\tan \beta_{1m} - \tan \beta_{2m}) \right]^{1/3}} \quad [\text{rpm}] \quad (35)$$

If ΔP_t is the rise of the total pressure in the stage, and η the total-to-total stage efficiency, there is

$$\frac{Q \Delta P_t}{\eta} = M\omega = \text{HP}(550) \quad (36)$$

With Eqs. 30, 31 and 3

$$\frac{\Delta P_t}{\eta} = \frac{M\omega}{Q} = \rho_o K\omega = \rho_o \omega^2 R_m^2 X \quad (37)$$

For^a driving power of 135 HP, the pressure rise ΔP_t is obtained directly from the second equality of Eq. 36. With Eq. 33,

$$\frac{\Delta P_t}{\eta} = 130.72 \frac{\tan\beta_{1m} + \tan\beta_{2m}}{(N/1000)} \quad [\text{lb/ft}^2]$$

or

$$\frac{\Delta P_t}{\eta} = 25.14 \frac{\tan\beta_{1m} + \tan\beta_{2m}}{(N/1000)} \quad [\text{in.H}_2\text{O}] \quad (38)$$

5. DETERMINATION OF BLADING PARAMETERS.

Most desirable for the present tests are bladings that produce the highest pressure rise at the lowest speed of rotation. Such bladings will also have low flow rates. Figures 4 and 5 show that the angles β_{1m} should be as high as possible for these conditions. Figure 6 indicates that the speed of rotation is not changing greatly with β_{1m} and that its magnitude is primarily a function of the diffusion factor D_m , which should be as high as possible.

Figure 7 shows the flow deflections $\Delta\beta = \beta_{1m} - \beta_{2m}$ for different angles β_{1m} and diffusion factors D_m at a solidity $\sigma_m = 1$, which establish the quantities X and Y of Eqs. 4 and 11, respectively. These values introduced into Eq. 12 show that the axial velocity V_{a2} becomes imaginary for β_{1m} greater than a particular angle β_{1m}^* which is depending on D_m . These limits of β_{1m}^* for $V_{a2} = 0$ are:

D_m	= 0.30	0.35	0.40	0.45
β_{1m}^*	= 45.7°	47.2°	47.8°	48.4°

Hence the flow angles β_{1m} cannot exceed about 45°.

Table I gives the flow conditions along the radius for $\beta_{1m} = 45^\circ$ and $D_m = 0.4$ at the mean radius R_m , as calculated by the previously established relations. It can be noted that the diffusion factor D_R at the rotor tip is in excess of 0.56, hence unacceptably high. For $\beta_{1m} = 45^\circ$ and $D_m = 0.35$ the resulting data are listed in Table II. Although D_R at the rotor tip is about 0.5, the large deceleration of the axial velocity components at the outer radius from $V_{a1}/\omega R_m = 0.4445$ to $V_{a2}/\omega R_m = 0.2629$, together with the negative flow deflection $\Delta\beta$ at this station, make this blading undesirable for the present purposes. The angle β_{1m} is therefore taken as 40° and Table III lists the data obtained for $D_m = 0.4$. For this value the diffusion factor D_R at the rotor tip is larger than 0.5. Repeating the calculations for $\beta_{1m} = 40^\circ$ and $D_m = 0.35$ gave the flow conditions of Table IV which seem acceptable in all respects. These parameters are therefore chosen for a more detailed investigation of the flow conditions by taking account of the radial shift of the stream surfaces from rotor inlet to rotor discharge with the resulting velocity diagram of Fig. 3.

6. METHOD FOR BETTER APPROXIMATION OF FLOW CONDITIONS.

The quantities listed in Table IV will be called the first approximation of the flow conditions. Figure 8 shows the velocity ratios $V_{a1}/(\omega R_m)$ and $V_{a2}/(\omega R_m)$ of this approximation as functions of the radius ratio R/R_m . These values were obtained by assuming that V_{a1} and V_{a2} at R_m are equal, and identical with the quantity V_m of Eqs. 8 and 9. Moreover a fluid particle entering the rotor at a radius R_1 is supposed to leave the rotor at the same radius, or $R_2 = R_1 = R$. This condition led to Eq. 1 which formulates that all fluid particles receive the same energy increase ΔH in the rotor. However if R_1 and R_2 differ, this requirement must be expressed by

$$\Delta H = \omega(R_2 V_{u2} - R_1 V_{u1}) = \text{Constant} \quad (39)$$

If it is assumed that the energy increase ΔH in the stage is that of the first approximation, there is from Eqs. 1 and 3

$$\Delta H = \omega K = \omega^2 R_m^2 X \quad (40)$$

where, from Table IV,

$$X = 0.36578$$

Now, if the tangential components V_{u1} along the radius R_1 ahead of the rotor are taken to be those of the first approximation, namely, by Eq. 5

$$\frac{V_{u1}}{\omega R_m} = -\frac{R_m}{R_1} \frac{X}{2} + \frac{1}{2} \frac{R_1}{R_m} \quad (41)$$

the distribution of V_{u2} along the radius R_2 after the rotor is from Eqs. 39, 40 and 41,

$$\frac{V_{u2}}{\omega R} = \frac{R_m}{R_2} \left[\frac{X}{2} + \frac{1}{2} \left(\frac{R_1}{R_m} \right)^2 \right] \quad (42)$$

Thus, to satisfy Eq. 39, the components V_{u2} can be determined only if the corresponding radii R_1 and R_2 of the stream surfaces are known. The results of the first approximation will be used to establish an approximate relationship between R_1 and R_2 by applying the equation of continuity. In particular, for a chosen value of R_1 the corresponding radius R_2 is found from the condition that the same mass flow rate has to occur between R_2 and the hub radius R_{2h} , as between the radius R_1 and the hub radius R_{1h} . The adopted procedure is explained in the next paragraph. With R_2 known as a function of R_1 , the distribution of the components V_{u2} in radial direction are obtained from Eq. 42. The axial components V_{a2} along R_2 can then be determined by solving Eq. 16 (41) of Ref. 2, or

$$\frac{\partial (V_{a2})^2}{\partial R_2} = -\frac{2 V_{u2}}{R_2} \frac{\partial (R_2 V_{u2})}{\partial R_2} \quad (43)$$

if the curvatures of the generatrices of the stream surfaces and radial entropy gradients are ignored. The data obtained in this manner from Eq. 43 represent the so-called second approximation. Applied to the equation of continuity they permit a check of the relationship between R_1 and R_2 . If this interdependence does not correspond to that obtained from the first approximation, it will be necessary to carry out further iterations.

Evidently, if curvature effects and entropy gradients are ignored, the distribution of V_{a1} along R_1 for V_{u1} of Eq. 41 is not affected by the radial shift from R_1 and R_2 , hence the flow conditions ahead of the rotor are those given in Table IV.

The relations for the second and, if necessary, further approximations, can be simplified by introducing the following dimensionless quantities:

$$\begin{aligned}
 r_1 &= \frac{R_1}{R_m} & ; & & r_2 &= \frac{R_2}{R_m} \\
 V_{a1}^* &= \frac{V_{a1}}{\omega R_m} & ; & & V_{a2}^* &= \frac{V_{a2}}{\omega R_m} \\
 V_{u1}^* &= \frac{V_{u1}}{\omega R_m} & ; & & V_{u2}^* &= \frac{V_{u2}}{\omega R_m} \\
 V_{m1}^* &= \frac{V_{m1}}{\omega R_m} & ; & & V_{m2} &= \frac{V_{m2}}{\omega R_m}
 \end{aligned} \tag{44}$$

Then, Eq. 42 rewritten is

$$V_{u2}^* = \frac{1}{2r_2} (X + r_1^2) \tag{45}$$

and Eq. 43 becomes

$$\frac{\partial (V_{a2}^*)^2}{\partial r_2} = - \left[\frac{X}{r_2^2} + \left(\frac{r_1}{r_2} \right)^2 \right] \frac{1}{2} \frac{\partial (X + r_1^2)}{\partial r_2}$$

Since X is a constant,

$$\frac{\partial (v_{a2}^*)^2}{\partial r_2} = - \left[\frac{x + r_1^2}{r_2^2} \right] r_1 \frac{\partial r_1}{\partial r_2} = - f(r_2) \quad (46)$$

Hence, if r_1 is known as a function of r_2 , the function $f(r_2)$ can be established. Denoting the as yet unknown value of v_{a2}^* at $r_2 = 1$ by v_{m2}^* , the integration of Eq. 46 gives

$$(v_{a2}^*)^2 = (v_{m2}^*)^2 - \int_1^{r_2} f(r_2) dr_2 \quad (47)$$

The quantity v_{m2}^* must be obtained by iterations with the help of the equation of continuity. Figure 4 shows that the pressure rise per stage does not exceed 16 inches of water. In a stage with 50 percent reaction the pressure rise in the rotor is then about 8 inches of water, giving a pressure ratio of about 1.02 for an inlet pressure of 14.7 psia. The ratio ρ_2/ρ_1 of the mass density after and ahead of the rotor will therefore not exceed a value of 1.014.

The volume flow rate Q_1 ahead of the rotor between the hub radius R_{1h} and an arbitrary radius R_1 is

$$Q_1 = 2\pi \int_{R_{1h}}^{R_1} R_1 v_{a1} dR_1$$

With the quantities of Eq. 44 let

$$Q_1 = 2\pi\omega R_m^3 Q_1^* \quad (48)$$

by introducing the dimensionless quantity

$$Q_1^* = \int_{R_{1h}/R_m}^{r_1} r_1 v_{a1}^* dr_1 \quad (49)$$

The total volume flow rate Q_{1max} ahead of the rotor is

$$Q_{1max} = 2\pi\omega R_m^3 Q_{1max}^* \quad (50)$$

where $Q_{1\max}^*$ is obtained from Eq. 49 for the upper limit $r_1 = R_{1t}/R_m$ of the integral, where R_{1t} is the tip radius.

Similarly, for the conditions after the rotor the volume flow rate Q_2 between R_{2h} and R_2 is

$$Q_2 = 2\pi\omega R_m^3 Q_2^* \quad (51)$$

$$\text{with } Q_2^* = \int_{R_{2h}/R_m}^{r_2} r_2 v_{a2}^* dr_2 \quad (52)$$

The total volume flow rate $Q_{2\max}$ at station 2 is

$$Q_{2\max} = 2\pi\omega R_m^3 Q_{2\max}^* \quad (53)$$

The quantity $Q_{2\max}^*$ is obtained from Eq. 52 for the upper limit $r_2 = R_{2t}/R_m$ of the integral.

The equality of the mass flow rates at stations (1) and (2) can be expressed by

$$\bar{\rho}_1 Q_{\max} = \bar{\rho}_2 Q_{2\max} k_B'$$

where $\bar{\rho}_1, \bar{\rho}_2$ are the average mass densities at (1) and (2), and k_B'

is the so-called blockage factor that takes account of the increased regions of reduced velocity in the so-called wall boundary layers near the hub and the tip at station (2), compared to those at station (1). Then, with Eqs. 50 and 53

$$Q_{2\max}^* = \frac{Q_{1\max}^*}{(\bar{\rho}_2/\bar{\rho}_1)k_B'} = \frac{Q_{1\max}^*}{k_B} \quad (54)$$

Experience shows that the blockage factor k_B' varies between 0.96 and 0.97. Hence, since $\bar{\rho}_2/\bar{\rho}_1 \approx 1.014$, the quantity $Q_{2\max}^*$ will be determined by Eq. 54 with a factor $k_B = (1.014)(0.965) = 0.98$ for the value of $Q_{1\max}^*$ obtained from Eq. 49. This value of $Q_{2\max}^*$ is used to

$$\frac{\partial (v_{a2}^*)^2}{\partial r_2} = - \left[\frac{x + r_1^2}{r_2^2} \right] r_1 \frac{\partial r_1}{\partial r_2} = - f(r_2) \quad (46)$$

Hence, if r_1 is known as a function of r_2 , the function $f(r_2)$ can be established. Denoting the as yet unknown value of v_{a2}^* at $r_2 = 1$ by v_{m2}^* , the integration of Eq. 46 gives

$$(v_{a2}^*)^2 = (v_{m2}^*)^2 - \int_1^{r_2} f(r_2) dr_2 \quad (47)$$

The quantity v_{m2}^* must be obtained by iterations with the help of the equation of continuity. Figure 4 shows that the pressure rise per stage does not exceed 16 inches of water. In a stage with 50 percent reaction the pressure rise in the rotor is then about 8 inches of water, giving a pressure ratio of about 1.02 for an inlet pressure of 14.7 psia. The ratio ρ_2/ρ_1 of the mass density after and ahead of the rotor will therefore not exceed a value of 1.014.

The volume flow rate Q_1 ahead of the rotor between the hub radius R_{1h} and an arbitrary radius R_1 is

$$Q_1 = 2\pi \int_{R_{1h}}^{R_1} R_1 v_{a1} dR_1$$

With the quantities of Eq. 44 let

$$Q_1 = 2\pi \omega R_m^3 Q_1^* \quad (48)$$

by introducing the dimensionless quantity

$$Q_1^* = \int_{R_{1h}/R_m}^{r_1} r_1 v_{a1}^* dr_1 \quad (49)$$

The total volume flow rate Q_{1max} ahead of the rotor is

$$Q_{1max} = 2\pi \omega R_m^3 Q_{1max}^* \quad (50)$$

where $Q_{1\max}^*$ is obtained from Eq. 49 for the upper limit $r_1 = R_{1t}/R_m$ of the integral, where R is the tip radius.

Similarly, for the conditions after the rotor the volume flow rate Q_2 between R_{2h} and R_2 is

$$Q_2 = 2\pi\omega R_m^3 Q_2^* \quad (51)$$

$$\text{with } Q_2^* = \int_{R_{2h}/R_m}^{r_2} r_2 v_{a2}^* dr_2 \quad (52)$$

The total volume flow rate $Q_{2\max}$ at station 2 is

$$Q_{2\max} = 2\pi\omega R_m^3 Q_{2\max}^* \quad (53)$$

The quantity $Q_{2\max}^*$ is obtained from Eq. 52 for the upper limit $r_2 = R_{2t}/R_m$ of the integral.

The equality of the mass flow rates at stations (1) and (2) can be expressed by

$$\bar{\rho}_1 Q_{\max} = \bar{\rho}_2 Q_{2\max} k_B'$$

where $\bar{\rho}_1, \bar{\rho}_2$ are the average mass densities at (1) and (2), and k_B'

is the so-called blockage factor that takes account of the increased regions of reduced velocity in the so-called wall boundary layers near the hub and the tip at station (2), compared to those at station (1). Then, with Eqs. 50 and 53

$$Q_{2\max}^* = \frac{Q_{1\max}^*}{(\bar{\rho}_2/\bar{\rho}_1)k_B'} = \frac{Q_{1\max}^*}{k_B} \quad (54)$$

Experience shows that the blockage factor k_B' varies between 0.96 and 0.97. Hence, since $\bar{\rho}_2/\bar{\rho}_1 \approx 1.014$, the quantity $Q_{2\max}^*$ will be determined by Eq. 54 with a factor $k_B = (1.014)(0.965) = 0.98$ for the value of $Q_{1\max}^*$ obtained from Eq. 49. This value of $Q_{2\max}^*$ is used to

check whether the distribution of V_{a2}^* along r_2 , obtained from Eq. 47 with a chosen value of V_{m2}^* , meets the continuity requirements. To this end the ratios V_{a2}^* , thus determined, are introduced into Eq. 52 and integrated from R_{2h}/R_m to R_{2t}/R_m . If the resulting quantity Q_{2max}^* differs from Q_{1max}^*/k_B of Eq. 54, the process must be repeated with a new value of V_{m2}^* until agreement is reached. However, the final distribution of V_{a2}^* along r_2 must also produce the same radial shift of the streamlines as was determined from the first approximation, and which was used to establish the function $f(r_2)$ of Eq. 46 and V_{a2}^* by Eq. 47. The equation of continuity is again used to obtain r_2 as function of r_1 from the new V_{a2}^* - distribution. If this relationship between r_2 and r_1 differs from the one obtained with the data of the first approximation, it must be used to establish a new function $f(r_2)$ of Eq. 46 which has to be introduced into Eq. 47 to calculate V_{a2}^* . This process requires the same iterations for V_{m2}^* as discussed above. The iterative procedure must be repeated until the functions r_2 vs. r_1 assumed for the determination of $f(r_2)$ and that obtained from the resulting profile of V_{a2}^* are identical.

7. RESULTS OF FLOW CALCULATIONS

Table V lists the ratios V_{a1}^* and V_{a2}^* of Table IV, and additional values at other radius ratios, which were used to calculate the quantities Q_1^* and Q_2^* in accordance with Eqs. 49 and 52, by integrations with the trapezoidal rule. It can be noted that the initial assumption $V_{m1} = V_{m2} = V_m$ does not satisfy continuity since Q_{2max}^* is $(0.9832)Q_{1max}^*$, whereas by Eq. 54 with $k_B = 0.98$ it should be $(1.0204)Q_{1max}^*$, or equal to 0.404947.

The ratios Q_1/Q_{1max} , and Q_2/Q_{2max} of Table V are plotted in Fig. 9. For the condition $Q_2/Q_{2max} = Q_1/Q_{1max}$, the radius ratio r_2 for a stream surface that has the radius ratio r_1 ahead of the rotor can be read on the upper horizontal scale of the figure. The relationship r_1 vs. r_2 thus obtained is shown in Fig. 10, which also gives a curve $\Delta r = (r_1 - r_2)$ as function of r_2 in a larger scale drawn through the data points. From this curve were obtained the derivatives $\partial(\Delta r)/\partial r_2$ by graphical means. Then

$$\frac{\partial r_1}{\partial r_2} = \frac{\partial(r_2 + \Delta r)}{\partial r_2} = 1 + \frac{\partial(\Delta r)}{\partial r_2} \quad (55)$$

The numerical values of these derivatives and of r_1 for different ratios r_2 are listed in Table VI, together with the function $f(r_2)$ of Eq. 46 for $X = 0.36578$.

The function $f(r_2)$ is plotted in Fig. 11. Interpolated values between the calculated points were used to integrate $f(r_2)$ with Simpson's rule, i.e. by placing a parabola through three neighboring points. The results of the integration in accordance with Eq. 47 are listed in Table VI and plotted in Fig. 11. Table VI also shows the calculated values of V_{a2}^* in accordance with Eq. 47, for several assumed values of V_{m2}^* , and the corresponding magnitudes of Q_2^* of Eq. 52 obtained by integrations with the trapezoidal rule. As pointed out earlier, the value of Q_2^* at $r_2 = 1.25$, which is denoted by $Q_{2\max}^*$, should equal 0.404947. For $V_{m2}^* = 0.838$ this condition is closely realized and the corresponding values of V_{a2}^* are taken to satisfy the continuity requirement. For $V_{m2}^* = 0.838$ the ratios $Q_2/Q_{2\max}$ are shown in Table VI also.

It must now be checked whether the relation between r_1 and r_2 from the first approximation of Fig. 9, which was used to calculate the velocity distributions of Table VI, agrees with the function r_2 vs. r_1 that is obtained from the second approximation. Since the change of V_{u1}^* along r_1 , hence of V_{a1}^* along r_1 , is the same for both approximations, this check can be made by plotting $Q_2/Q_{2\max}$ as function of r_2 for both sets of data. Figure 12 shows a curve of $Q_2/Q_{2\max}$ vs. r_2 drawn through the values of Fig. 9 which are marked by crosses. The data points given by circles in Fig. 12 represent the values of Table VI for $V_{m2}^* = 0.838$. It can be noted that these points lie very closely on the curve of the first approximation, hence additional iterations are unnecessary, and the data inside the heavily framed portion of Table VI can be considered to represent the actual flow conditions after the rotor for the set of initial assumptions; that is, if curvature effects and entropy gradients are ignored. This final distribution of V_{a2}^* is also shown by the dash-dotted curve in Fig. 8.

Table VII summarizes the flow properties ahead of and after the rotor in accordance with the data of Table VI. Since the velocity diagram of the blading is that of Fig. 3, there are, with Eq. 41

$$V_{u1}^* = \frac{V_{u1}}{\omega R_m} = -\frac{X}{2r_1} + \frac{r_1}{2} \quad (56)$$

and

$$W_{u1}^* = \frac{W_{u1}}{\omega R_m} = \frac{\omega R_1 - V_{u1}}{\omega R_m} = r_1 - V_{u1}^* = \frac{X}{2r_1} + \frac{r_1}{2} \quad (57)$$

Thus

$$\tan \alpha_1 = \frac{V_{u1}^*}{V_{a1}} \quad (58)$$

and

$$\tan \beta_1 = \frac{W_{u1}^*}{V_{a1}^*} \quad (59)$$

With Eq. 45 and from Fig. 3

$$W_{u2}^* = \frac{W_{u2}}{\omega R_m} = \frac{\omega R_2 - V_{u2}}{\omega R_m} = r_2 - V_{u2}^* \quad (60)$$

and

$$\tan \alpha_2 = \frac{V_{u2}^*}{V_{a2}} \quad (61)$$

$$\tan \beta_2 = \frac{W_{u2}^*}{V_{a2}^*} \quad (62)$$

Also

$$V_1^* = \frac{V_1}{\omega R_m} = \frac{V_{a1}^*}{\cos \alpha_1} \quad (63)$$

$$W_1^* = \frac{W_1}{\omega R_m} = \frac{V_{a1}^*}{\cos \beta_1} \quad (64)$$

$$V_2^* = \frac{V_2}{\omega R_m} = \frac{V_{a2}^*}{\cos \alpha_2} \quad (65)$$

$$W_2^* = \frac{W_2}{\omega R_m} = \frac{V_{a2}^*}{\cos \beta_2} \quad (66)$$

Table VII also lists the NASA diffusion factors of Eqs. 17 and 18, where the subscripts o in $(D_R)_o$ and $(D_S)_o$ indicate that these factors are for a solidity $\sigma = \sigma_m = 1$ at $R/R_m = 1$, and that the chord of the rotor and stator blades does not change in radial direction. In actuality the diffusion factors should be calculated for the conditions along the stream surfaces by taking account of their radial shift. Then, by Ref. 3, for the rotor

$$D_R = 1 - \frac{W_2}{W_1} + \frac{R_1 W_{u1} - R_2 W_{u2}}{\sigma_R (R_1 + R_2) W_1} \quad (67)$$

A similar expression is defined in Ref. 3 for stators. However, in the present case where the radial shift is slight, the relations of Eqs. 17 and 18 can be used with very small error. In fact, the differences are so insignificant that the blade profiles in stator and rotor will be determined for cylindrical stream surfaces, with the actual flow angles of Table VII, instead of along the actual stream surfaces. This procedure avoids the difficulties associated with the definition of the blade shapes for manufacturing purposes. If the profiles are considered to lie on arbitrary stream surfaces, the resulting blade shapes must later be intersected by cylinders, or by planes tangent to them, to be able to obtain profile coordinates that can be used and checked during manufacture with reasonable cost. The available calculating methods for the profile shapes and their angular orientation are not accurate enough to warrant the difficulties associated with the afore-mentioned procedure, particularly not because the theoretical, axisymmetric, stream surfaces are very nearly cylindrical in the present case. It must also be recognized that the actual stream surfaces in a blade row are not axisymmetric, hence particles that move along the two sides of a blade travel along different paths with different radii so that the flow

does not have the same two-dimensional character that exists in rectilinear cascade test rigs which were used to establish the profile design data.

In addition to, or as a replacement of the diffusion factor, the so-called equivalent diffusion ratio D_{EQ} is frequently used to establish the blade loading limits and the profile losses, especially if the flow incidence angles differ from those for minimum losses. By Ref. 4, for rotors at the design incidence angle, and for flows with varying through-flow components on cylindrical stream surfaces,

$$D_{EQR} = \frac{W_1}{W_2} \left[1.12 + \frac{0.61}{\sigma_R} \cos^2 \beta_1 \frac{2\Delta W_u}{V_{a1} + V_{a2}} \right] \quad (68)$$

For stators at the same conditions,

$$D_{EQS} = \frac{V_2}{V_1} \left[1.12 + \frac{0.61}{\sigma_S} \cos^2 \alpha_2 \frac{2\Delta V_u}{V_{a1} + V_{a2}} \right] \quad (69)$$

Experience has shown that the quantities D_{EQ} should not exceed values of about 1.7 since incipient flow separation is likely to occur for D_{EQ} between 2.0 and 2.2. The subscripts o in $(D_{EQR})_o$ and $(D_{EQS})_o$

of Table VII indicate that $\sigma_R = \sigma_S = 1$ at the mean radius R_m and that the blades have constant chord at all radii.

8. BLADE PROFILES AND STAGGER ANGLES

The thickness distribution of Fig. 13 will be used for the profiles of the stator and rotor blades. It corresponds to a C.4 base profile (See Ref. 5), except for the portions near the entrance which were thinned to arrange a smaller leading edge radius. The camber line of the profile is supposed to be a circular arc. Figure 14 shows the method that will be adopted to determine the profile coordinates for known camber angles α and maximum thickness ratios t/c . The orientation of the blade profiles in the cascade is fixed by the stagger angle γ as indicated in Fig. 14.

The profile data are determined with the two-dimensional method of chapter VI of Ref. 6, without taking account of the corrections for three-dimensional effects that are given in chapter VII of Ref. 6. It is felt that these corrections are not sufficiently well supported by experiments. Moreover, since the blades in the test compressor can be set at different angles the twist of the profiles along the blade height is more important than the stagger angle per se.

The camber angle ϕ for a rotor profile is obtained from

$$\phi = \frac{\Delta\beta - i_o + \delta_o}{1 - m + n} \quad (70)$$

where

$$\Delta\beta = \beta_1 - \beta_2$$

$$i_o = (K_i)_{sh} (K_i)_t (i_o)_{10} \quad (71)$$

$$\delta_o = (K_\delta)_{sh} (K_\delta)_t (\delta_o)_{10} \quad (72)$$

$$(i_o)_{10} = \text{zero camber incidence angle for } t/c = 0.1$$

$$(\delta_o)_{10} = \text{zero camber deviation angle for } t/c = 0.1$$

The recommended value of the correction factors $(K_i)_{sh}$ and $(K_\delta)_{sh}$ is 1.1 for C.4 profiles. The design incidence angle i for minimum loss is

$$i = i_o + n\phi \quad (73)$$

and the deviation angle δ for these conditions is

$$\delta = \delta_o + m\phi \quad (74)$$

The quantities $(i_o)_{10}$, $(\delta_o)_{10}$, m , and n are functions of the inlet flow angle β_1 and the blade solidity σ . The correction factor $(K_i)_t$ and $(K_\delta)_t$ depend on the thickness ratio t/c only. The blade stagger

angle γ of Fig. 14 is equal to

$$\gamma = \beta_1 - i - \frac{\phi}{2} \quad (75)$$

The stator blade profiles are obtained with Eqs. 70 to 74 also if β_1 is replaced by α_2 , and $\Delta\beta$ by $\Delta\alpha = \alpha_2 - \alpha_1$. The blade stagger angle is then

$$\gamma = \alpha_2 - i - \frac{\phi}{2} \quad (76)$$

For particular conditions the method of Ref. 6 gives excessively large negative incidence angles. Although negative incidence angles are beneficial for good stall margins, they may cause increased losses at the design point because of separations on the pressure side near the leading edge, especially if the latter has a small radius. If the above-mentioned situation occurs, reduced negative incidence angle i' will be assumed. Reference 6 shows that for these conditions the corresponding deviation angle δ is obtained from

$$\delta' = \delta + (i' - i) \left(\frac{d\delta}{di} \right) \quad (77)$$

where the function $(d\delta/di)$ depends on solidity and inlet flow angle. For a stator blade profile, for instance, there is

$$\Delta\alpha = \phi + i - \delta = \phi' + i' - \delta'$$

where the first expression on the right-hand side holds for the design conditions obtained with the usual approach, whereas the second holds for an assumed incidence angle i' from which the necessary blade camber angle ϕ' can be determined. With Eq. 77

$$\phi' = \Delta\alpha - i' + \delta' = \Delta\alpha - i' + \delta + (i' - i) \left(\frac{d\delta}{di} \right) \quad (78)$$

For this modified camber, the blade stagger angle γ' of a stator is
$$\gamma' = \alpha_2 - i' - \frac{\phi}{2} \quad (79)$$

The axial distances between the blade rows of the compressor are 2.875 inches. The axial clearance between the rows must be about 0.5 in. to provide room for flow survey probes, so that the axial blade widths must be less than about 2.4 inches. Further, since each rotor has 30 and each stator 32 blades, the solidities of the rotor and the

stator cascades must remain within certain limits.

The diffusion factors D and the equivalent diffusion ratios D_{EQ} of Table VII are measures for the aerodynamic loading of the blade profiles in a cascade. Excessive loading is associated with flow separations, increased losses, and reduced efficiencies. At low blade loadings the efficiency of the cascade is decreased also because of large boundary layer losses. Optimum efficiencies of a row of blades are obtained if at all radii the blade profiles have the highest possible loading and the same margin of safety against stalling. The choice of the latter depends on the operating characteristics of the system for which the compressor is used.

Table VII shows that this condition is not realized in symmetrical bladings with blades of constant chord. The diffusion factor of the rotor is highest at the outer radius and very much below permissible limits at the hub radius. The opposite trend would give a better performance since it is known that for the same diffusion factor the losses are higher at the rotor tip than at the hub. The data for the stator in Table VII show that the profiles are lightly loaded at the outer radius and have increased loading at the hub radius. For stators with radial clearance gaps at the hub radius the loading should be reversed also to obtain optimum performance, since the stator losses near the hub radius are larger than those at other radii even if the diffusion factor were constant everywhere along the blade.

Within the limitations imposed by the rotor and stator blade numbers and the necessity of maintaining sufficiently large axial blade clearances for the arrangement of flow survey probes, the blade chords of rotor and stator will be varied along the radius to slightly improve the distributions of the diffusion factors of Table VII. The chosen chords for the rotor blades are given in Table VIII, those for the stator blades in Table IX. From Table VIII it can be seen that, whereas for constant chord the factor D_R for the rotor changed from 0.2062 to 0.4376 from hub to tip, it now has values of 0.2498 and 0.4274 at these respective stations. The stator solidities obtained with the chosen chords are generally lower than those for constant chord in Table VII, which gave variations of D_S from 0.4107 to 0.3326 from hub to tip. By Table IX these respective values are 0.4240 and 0.3762 for the stator blades

with varying chord. These data show that with the symmetrical blading of Fig. 3 the earlier-mentioned optimum blade loadings cannot be achieved by varying the blade chord. Symmetrical bladings produce favorable conditions as far as the uniformity of the inlet flow velocities along the radius is concerned. Table VII shows for instance that the relative rotor inlet velocity W_1 decreases from 100 percent at the hub to 95.5 percent at the tip. The stator inlet velocity V_2 decrease in radial direction also, namely, from 100 percent at the hub to 81 percent at the outer radius. The actual magnitudes of W_1 and V_2 are nearly equal. Hence, in contrast to other blading types, symmetrical bladings are not Mach number limited by the conditions at the outer radius. For a given maximum Mach number, they can operate at higher peripheral speeds than other bladings and are therefore capable of producing a higher pressure rise per stage.

The best design of a blading would however be one that has uniform and equal velocities W_1 and V_2 along the radius, which are small with respect to the peripheral speed, and where the diffusion factors for all profiles are as high as the necessary stall margins permit. Investigations whether such bladings are possible will be carried out with a new computer program that is based on the calculating approach of this report, but with the additional refinement that the effect of the different losses along the radius will be taken into account.

The profile thicknesses of Table VIII and IX were chosen for practical and aerodynamic reasons. For the rotor the blade thickness increases about parabolically from tip to hub to obtain reduced centrifugal stresses, and the thickness of the stator blades varies about linearly to obtain small ratios t/c at the hub radius, and sufficient blade thickness at the outer radius for the attachment of the blade to its root section with the circular platform that is shown in Fig. 9 of Ref.1.

Whereas the incidence angles i of the rotor blade profiles of Table VIII seem reasonable, excessively large negative incidence angle are obtained with the design method of Ref. 6 for the stator profiles of Table IX at large radii. The data inside the heavily framed portion of Table IX are therefore replaced by those for the assumed incidence angles i' . The last lines in Tables VIII and IX give the axial widths of the rotor and stator blade profiles. For an axial spacing of 2.875 in.

of the stator and rotor rows, the average axial clearance between the rows is $2.875 - (1_{axr} + 1_{axs})/2$. Hence this axial clearance is about 0.47 in. at the hub radius and 0.76 in. at the outer radius. These clearances are large enough for blades with the usual radial stacking line through the centers of gravity of the different profiles. It must however be examined whether for blades with swept-back leading edges, which are to be tested during the program, the blade chords of Tables VIII and IX permit to arrange inter-stage instrumentation.

The data of Tables VIII and IX will be used to establish the profile coordinates with the relations of Fig. 14. For each profile there must be determined its cross-sectional area, the coordinates of the center of gravity and the angles of the principal axes with respect to the reference line of Fig. 14, and the minimum and maximum moments of inertia. These quantities are necessary to establish the stresses and the vibratory characteristics for particular three-dimensional arrangements of the profiles in the build-up of the blades.

9. COMPRESSOR PERFORMANCE.

If the losses in the inlet guide vanes of the compressor are ignored, the total pressure P_{t1} ahead of the first rotor at station (1) is everywhere constant. However the static pressure p_1 at station (1) changes along the radius, since for incompressible flows

$$p_1 = P_{t1} - \frac{\rho}{2} v_1^2 \quad (80)$$

At station (2) after the rotor the total and static pressures are denoted by P_{t2} and p_2 , where

$$p_2 = P_{t2} - \frac{\rho}{2} v_2^2 \quad (81)$$

In contrast to prior usage in this report the region after the stator is called station (3). At this station exist the same velocities and flow angles as at station (1) but the pressures differ from those at (1), and the static and total pressures p_3 and P_{t3} are related by

$$p_3 = P_{t3} - \frac{\rho}{2} v_1^2 \quad (82)$$

From Eq. 7(44) of Ref. 2 the conditions along a streamline in the rotor between stations (1) and (2) can be expressed by

$$p_2 + \frac{\rho}{2} W_2^2 - \frac{\rho}{2} \omega^2 R_2^2 = p_1 + \frac{\rho}{2} W_1^2 - \frac{\rho}{2} \omega^2 R_1^2 - \Delta P_R \quad (83)$$

where ΔP_R represents the losses in the rotor. In Eq. 83 the differences in the geopotential energies of the fluids are ignored. It is customary (See Ref. 6) to express the rotor losses by

$$\Delta P_R = Y_R \frac{\rho}{2} W_1^2 \quad (84)$$

In Ref. 3 the pressure loss coefficient Y_R is related to the diffusion factor D_R and the rotor solidity σ_R by

$$Y_R = \frac{2\sigma_R}{\cos\beta_2} \left[0.004 + 0.0639(D_R + 0.1)^{2.91} + 0.228D_R^{2.02} (1-\lambda_R)^{3.77} \right] \quad (85)$$

where

$$\lambda_R = \frac{R_{2t} - R_2}{R_{2t} - R_{2h}} = \frac{r_{2t} - r_2}{r_{2t} - r_{2h}} \quad (86)$$

The subscripts t and h refer to the outer and hub radius, respectively. Equation 85 is an approximate mathematical formulation of the empirical loss correlations of Ref. 6 which attempt to account for tip clearance effects with the factor λ_R , which increases the losses toward the rotor blade tip.

In Ref. 6 the loss in total pressure in the stator blade row is defined by

$$\Delta P_S = P_{t2} - P_{t3} = Y_S \frac{\rho}{2} V_2^2 \quad (87)$$

where by Ref. 3 the stator loss coefficient Y_S can be approximated by

$$Y_S = \frac{2\sigma_S}{\cos\alpha_1} \left[0.004 + 0.0639 (D_S + 0.1)^{2.91} + 0.057 D_S^{2.02} (1 - \lambda_S)^{3.77} \right] \quad (88)$$

The quantities σ_S and D_S are the stator solidity and the stator diffusion factor, respectively, and

$$\lambda_S = \frac{R_1 - R_{1h}}{R_{1t} - R_{1h}} \quad (89)$$

Due to the ratio λ_S the loss coefficients Y_S of Eq. 88 increase toward the hub radius where the stator blades have a radial clearance gap. Evidently Eqs. 84 and 88 do not take proper account of the tip clearance and end losses. A discussion in Section 13.7 of Ref. 2 shows that these losses must depend on the blade loading, the ratio of tip clearance and blade height, and the ratio of blade spacing and blade height, in addition to solidity. The present research program endeavors to establish a better and more physically correct evaluation of the end losses and tries to investigate whether peculiar blade shapes can be used to minimise their detrimental effects on stage performance.

From Fig. 3

$$W_1^2 = (\omega R_1 - V_{u1})^2 + V_{a1}^2 = \omega^2 R_1^2 - 2\omega R_1 V_{u1} + V_1^2$$

$$W_2^2 = (\omega R_2 - V_{u2})^2 + V_{a2}^2 = \omega^2 R_2^2 - 2\omega R_2 V_{u2} + V_2^2$$

Introduced in Eq. 83, and with Eq. 84,

$$P_2 + \frac{\rho V_2^2}{2} = P_1 + \frac{\rho V_1^2}{2} + \rho\omega(R_2 V_{u2} - R_1 V_{u1}) - Y_R \frac{\rho W_1^2}{2}$$

With Eqs. 39, 40, 80, and 81

$$P_{t2} - P_{t1} = \rho \omega^2 R_m^2 X - Y_R \frac{\rho}{2} W_1^2 \quad (90)$$

where X is a constant, equal to 0.36578, for the bladings under consideration. With Eq. 64 the above relations, rewritten, gives

$$\Pi_{t2} = \frac{P_{t2} - P_{t1}}{\rho \omega^2 R_m^2} = X - \frac{Y_R}{2} (W_1^*)^2 \quad (91)$$

From Eq. 87, with Eq. 65

$$\frac{P_{t2} - P_{t3}}{\rho \omega^2 R_m^2} = \frac{Y_S}{2} (V_2^*)^2 \quad (92)$$

and

$$\Pi_{t3} = \frac{P_{t3} - P_{t1}}{\rho \omega^2 R_m^2} = X - \frac{Y_R}{2} (W_1^*)^2 - \frac{Y_S}{2} (V_2^*)^2 \quad (93)$$

For frictionless conditions the coefficients Y_R and Y_S would be zero and the theoretical total pressure rise in the stage would be $X \rho \omega^2 R_m^2$ by Eq. 93. Hence the total-to-total stage efficiency becomes

$$\eta_t = 1 - \frac{Y_R (W_1^*)^2 + Y_S (V_2^*)^2}{2X} \quad (94)$$

Equation 94 should actually be evaluated along particular stream surfaces; that is, W_1^* and V_2^* should be taken at the corresponding radii R_1 and R_2 of a stream surface. However, because of the small radial shift of the flow paths in the stage, and for the other reasons explained in paragraph 8, Eq. 94 will be determined for cylindrical stream surfaces and constant radii $R = R_1 = R_2$. A volume flow averaged stage efficiency $\overline{\eta}_t$ can be evaluated from the different efficiencies

η_t along the radius by

$$\bar{\eta}_t = \frac{\int_{r_{1h}}^{r_{1t}} dQ_1 \eta_t}{\int_{r_{1h}}^{r_{1t}} dQ_1} \quad (95)$$

With Eqs. 49 and 50, with $r_{1h} = R_{1h}/R_m$, $r_{1t} = R_{1t}/R_m$,

$$\bar{\eta}_t = \frac{\int_{r_{1h}}^{r_{1t}} r_1 v_{a1}^* \eta_t dr_1}{Q_{1max}^*} \quad (96)$$

where from Table V, $Q_{1max}^* = 0.397007$. The average total pressure \bar{P}_{t3} at station 3 is then obtained from

$$\bar{\Pi}_{t3} = \frac{\bar{P}_{t3} - P_{t1}}{\rho \omega^2 R_m^2} = \bar{\eta}_t \times \quad (97)$$

In addition to the changes of the total pressures P_{t2} and P_{t3} in radial direction, the distributions of the static pressures p_1 , p_2 , and p_3 are of interest also for comparisons with the measured pressures and for the determination of the aerodynamic blade forces. From Eqs. 80 and 63

$$\Pi_1 = \frac{p_1 - P_{t1}}{\rho \omega^2 R_m^2} = -\frac{1}{2} (v_1^*)^2 \quad (98)$$

From Eqs. 81, 91, and 65

$$\pi_2 = \frac{p_2 - p_{t1}}{\rho \omega^2 R_m^2} = X - \frac{Y_R}{2} (W_1^*)^2 - \frac{1}{2} (V_2^*)^2 \quad (99)$$

From Eqs. 82, 93, and 63

$$\pi_3 = \frac{p_3 - p_{t1}}{\rho \omega^2 R_m^2} = X - \frac{Y_R}{2} (W_1^*)^2 - \frac{Y_S}{2} (V_2^*)^2 - \frac{1}{2} (V_1^*)^2 \quad (100)$$

Table X shows the loss coefficients in rotor and stator for the solidities of Tables VIII and IX, and gives the dimensionless pressure coefficients. It can be noted that highest efficiency $\eta_t = 0.9422$ occurs at the mean radius, whereas the efficiencies at the tip and the hub are 0.8238 and 0.8857, respectively. The last line of Table X lists the integrands of Eq. 96. By trapezoidal integration the integral of Eq. 96 is 0.363035, hence the average stage efficiency is

$$\eta_t = \frac{0.363035}{0.397007} = 0.9144$$

Then, by Eq. 97, with $X = 0.36578$

$$\bar{\pi}_{t3} = \frac{\bar{p}_{t3} - p_{t1}}{\rho \omega^2 R_m^2} = 0.33448$$

Figure 15 is a plot of the pressure coefficients of Table X to show the pressure changes along the radius of the stage. It can be noted that equal increases of the static pressures in rotor and stator occur at a radius ratio $r = R/R_m$ of about 1.1. At smaller radii most of the static pressure rise occurs in the stator, whereas the opposite occurs between $r = 1.1$ and the tip. Hence because of the resulting radial velocity distributions a symmetric blading does not have a constant degree of reaction of 50 percent.

With the established data the operating conditions of the compressor can be determined so that 135 HP are absorbed by one, two, or three stages. By Eqs. 30, 31, 40, and 50, the driving moment M for one stage is

$$M = Q_{1\max} \rho_o \omega R_m^2 X = 2\pi \omega^2 R_m^5 \rho_o Q_{1\max}^* X$$

where $Q_{1\max}^* = 0.397007$ from Table V. Then, by Eq. 36, for 135 HP, $R_m = 1.2$ ft, and $\rho_o = 2.371(10^{-3})$ slug/ft³,

$$\omega^3 = \frac{(135)(550)}{2\pi \rho_o R_m^5 Q_{1\max}^* X} = 13.793 (10^6)$$

or

$$\omega = 239.82 \text{ radians/sec}$$

and
$$N = \frac{30}{\pi} \omega = 2290 \text{ rpm}$$

Then, by Eq. 50,

$$Q_{1\max} = 1034 \text{ ft}^3/\text{sec}$$

From Eq. 97 with $X = 0.36578$ and $\bar{\eta}_t = 0.9114$

$$\bar{P}_{t3} - P_{t1} = \rho_o \omega^2 R_m^2 X \bar{\eta}_t = 62.4 \text{ lb/ft}^2 = 12.6 \text{ in. H}_2\text{O}$$

The following table gives these data if one, two, or three stages absorb a driving power of 135 HP.

		One Stage	Two Stages	Three Stages
		Z = 1	Z = 2	Z = 3
Speed N	(rpm)	2290	1818	1588
Volume Flow Rate $Q_{1\max}$	(ft ³ /sec)	1034	820	718
Pressure Rise $Z(\bar{P}_{t3} - P_{t1})$	(in. H ₂ O)	12.6	15.9	18.1
Rotor Tip Speed ωR_t	(ft/sec)	359.7	285.5	249.4

Before deciding on a drive for all three speeds, a study will be undertaken to determine the blade and rotor stresses, and to investigate whether critical shaft speeds and bearing capacities permit trouble-free operation for all cases.

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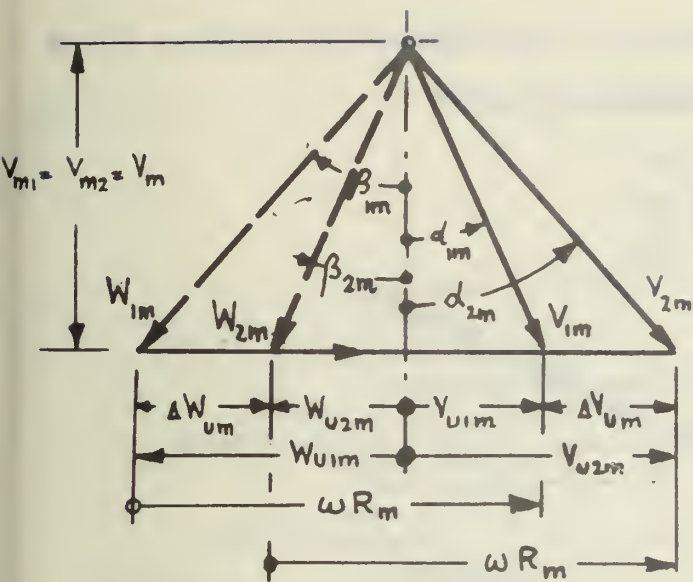


FIG. 1 VELOCITY DIAGRAM OF SYMMETRICAL STAGE AT MEAN RADIUS R_m

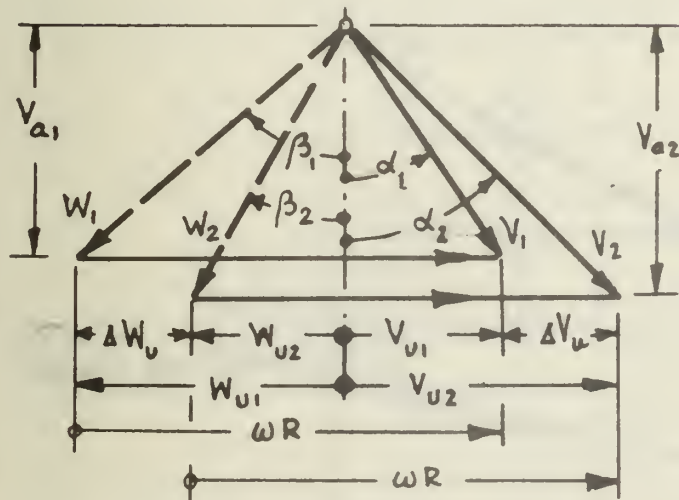


FIG. 2 VELOCITY DIAGRAM OF SYMMETRICAL STAGE FOR FLOW ON CYLINDRICAL STREAM SURFACE WITH $R_2 = R_1$

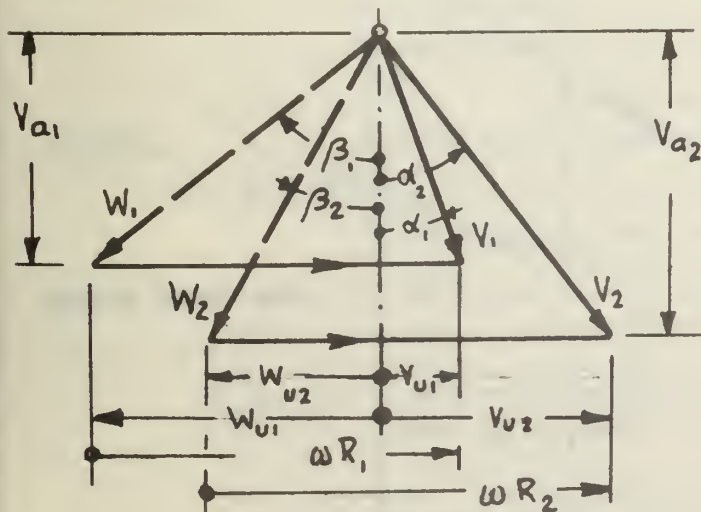


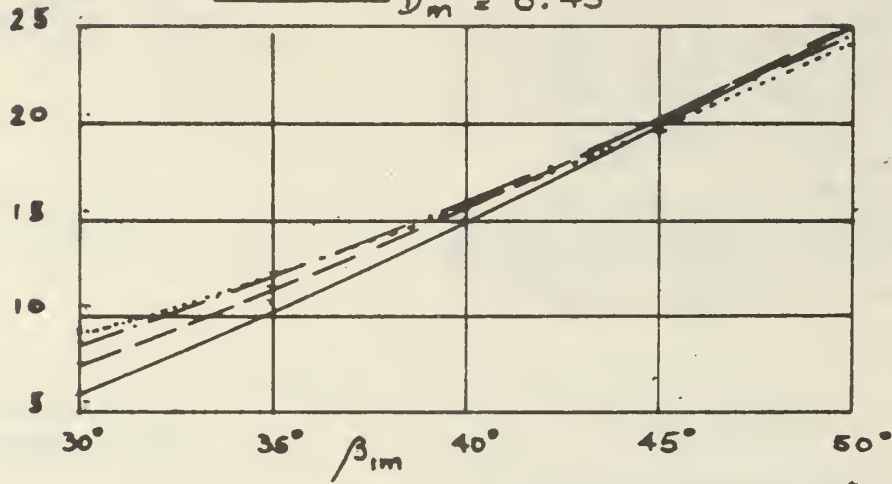
FIG. 3 VELOCITY DIAGRAM OF STAGE WITH FLOW ON ARBITRARY AXISYMMETRIC STREAM SURFACE

FIG. 4 THEORETICAL PRESSURE RISE $\Delta P_t / \eta$ IN COMPRESSOR FOR POWER OF 135 HP

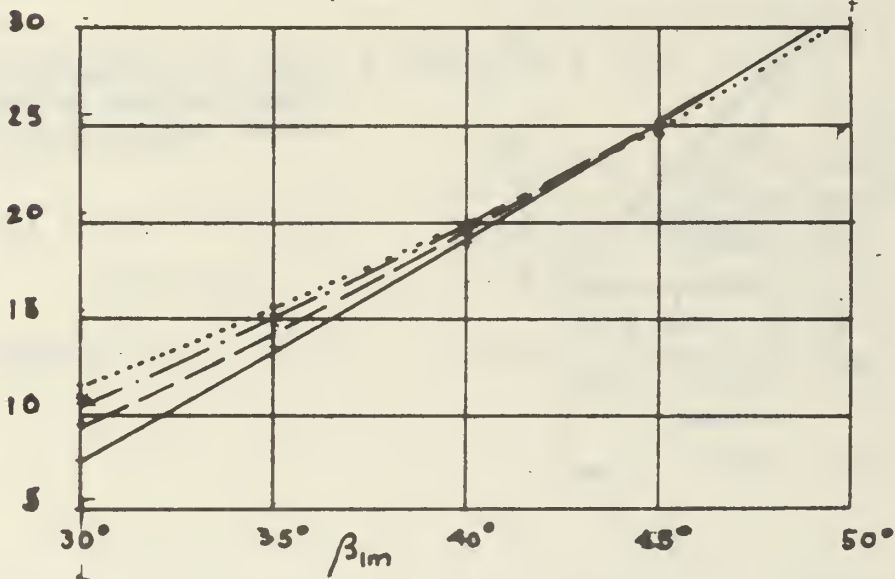
SYMMETRICAL BLADING WITH $\sigma_m = 1$

..... $D_m = 0.3$
 - - - - - $D_m = 0.35$
 - - - - - $D_m = 0.4$
 ————— $D_m = 0.45$

$\frac{\Delta P_t}{\eta}$
 (in. H₂O)



$\frac{\Delta P_t}{\eta}$
 (in. H₂O)



$\frac{\Delta P_t}{\eta}$
 (in. H₂O)

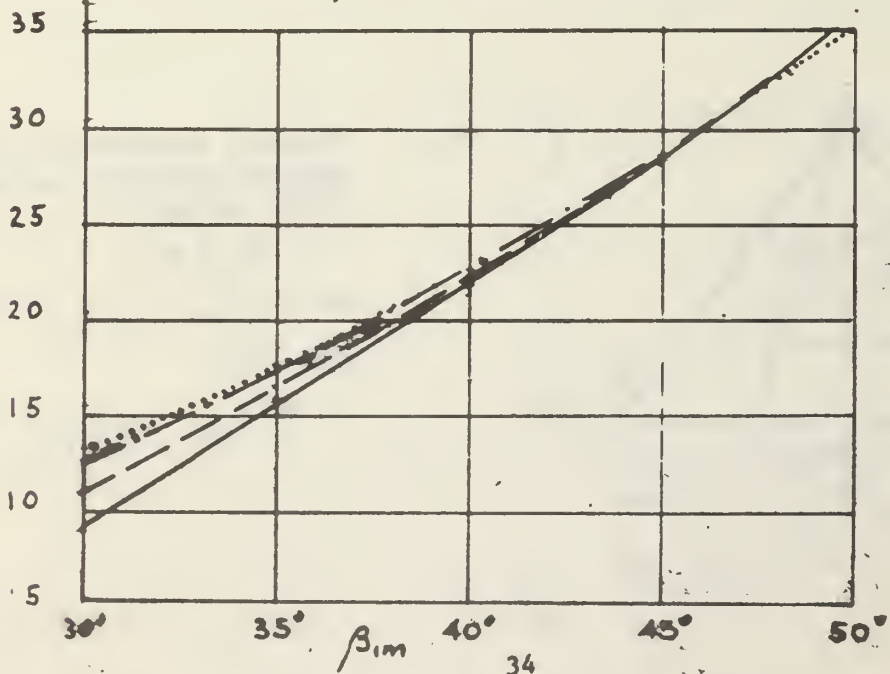


FIG. 5 VOLUME FLOW RATE Q OF COMPRESSOR FOR POWER OF 135 HP

SYMMETRICAL BLADING WITH $\sigma_m = 1$

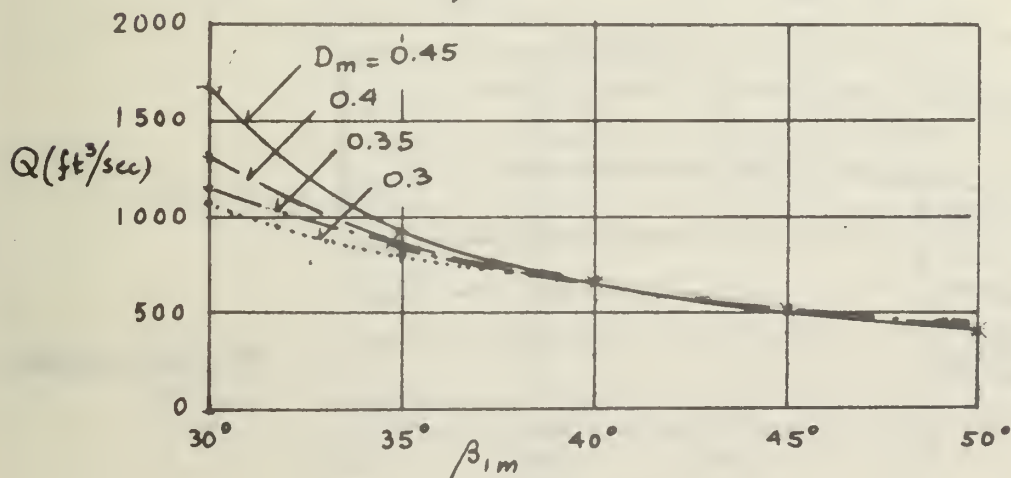
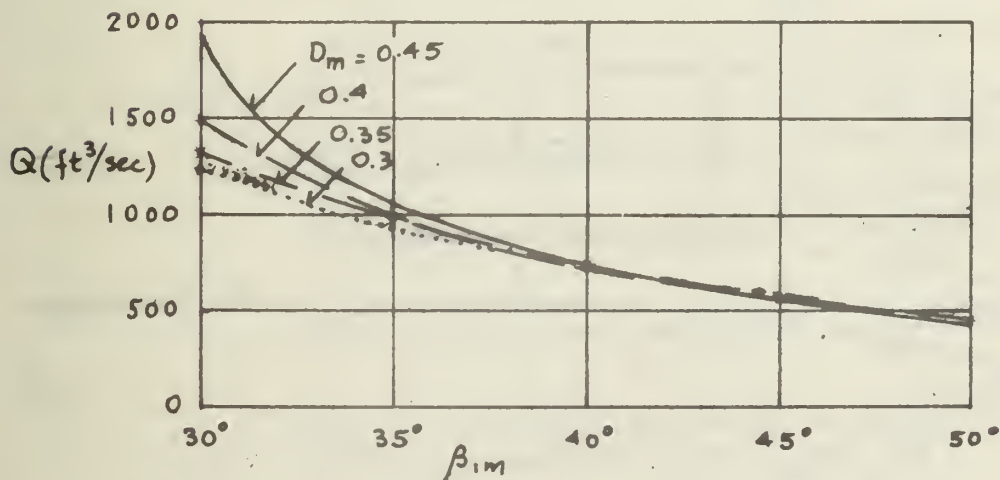
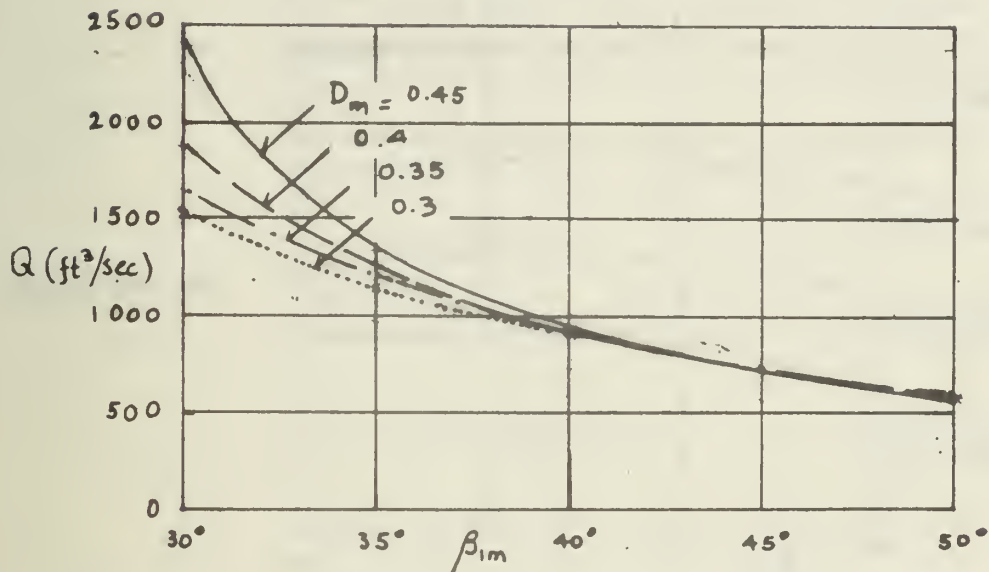


FIG.6 SPEED OF ROTATION N OF COMPRESSOR FOR POWER OF 135 HP

SYMMETRICAL BLADING WITH $\sigma_m = 1$

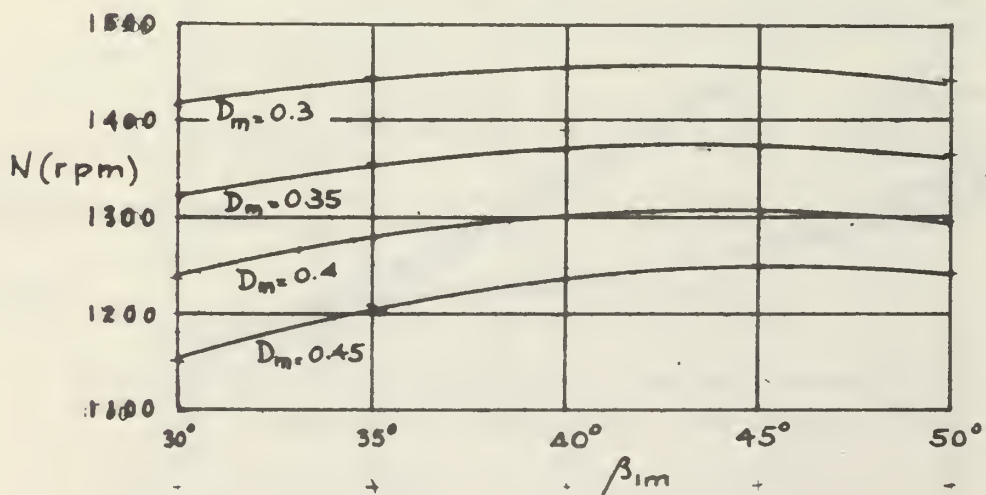
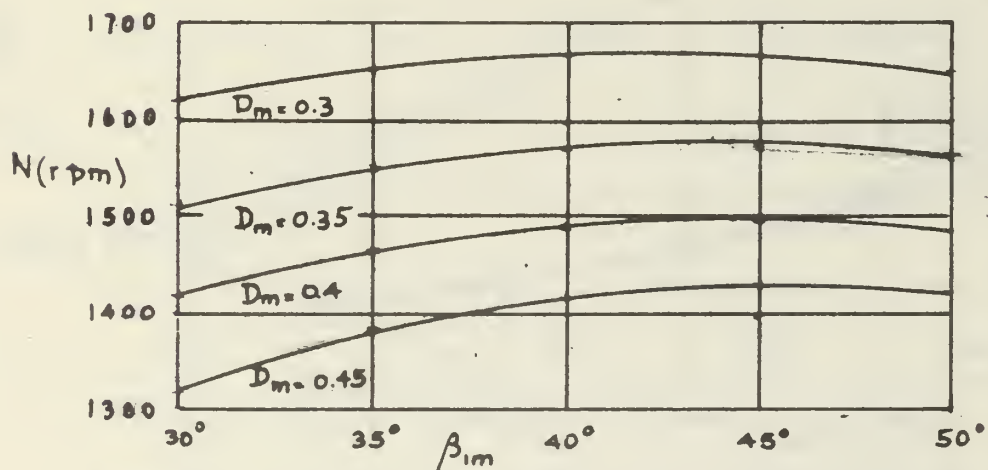
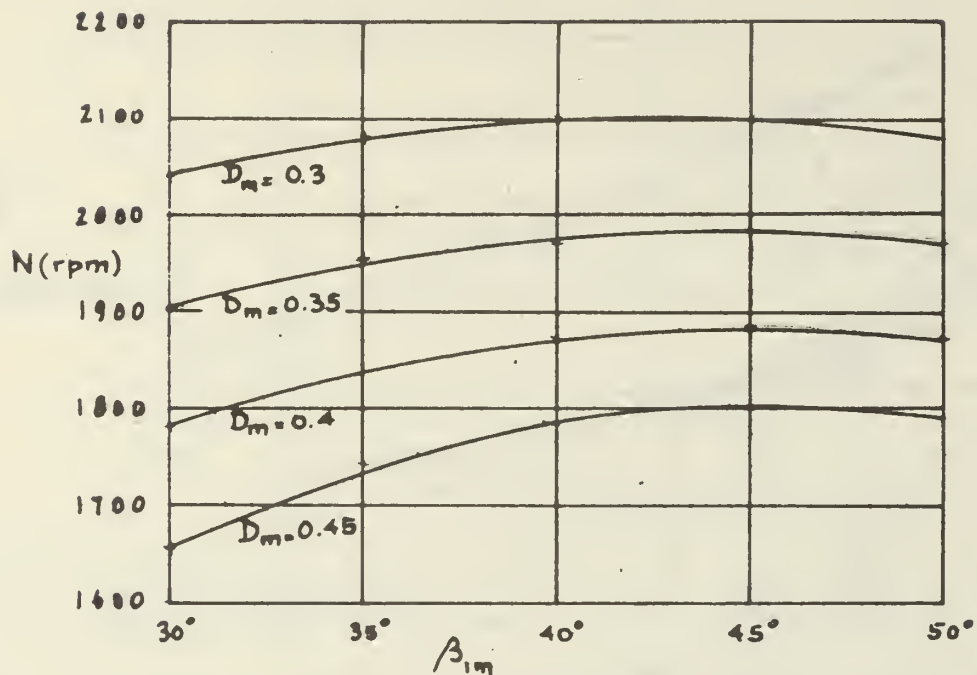


FIG. 7 FLOW DEFLECTIONS IN ROTOR CASCADE AT DIFFERENT VALUES
OF FLOW INLET ANGLE β_{1m} AND NASA DIFFUSION FACTOR D_m
AT BLADE SOLIDITY $\sigma_m = 1$

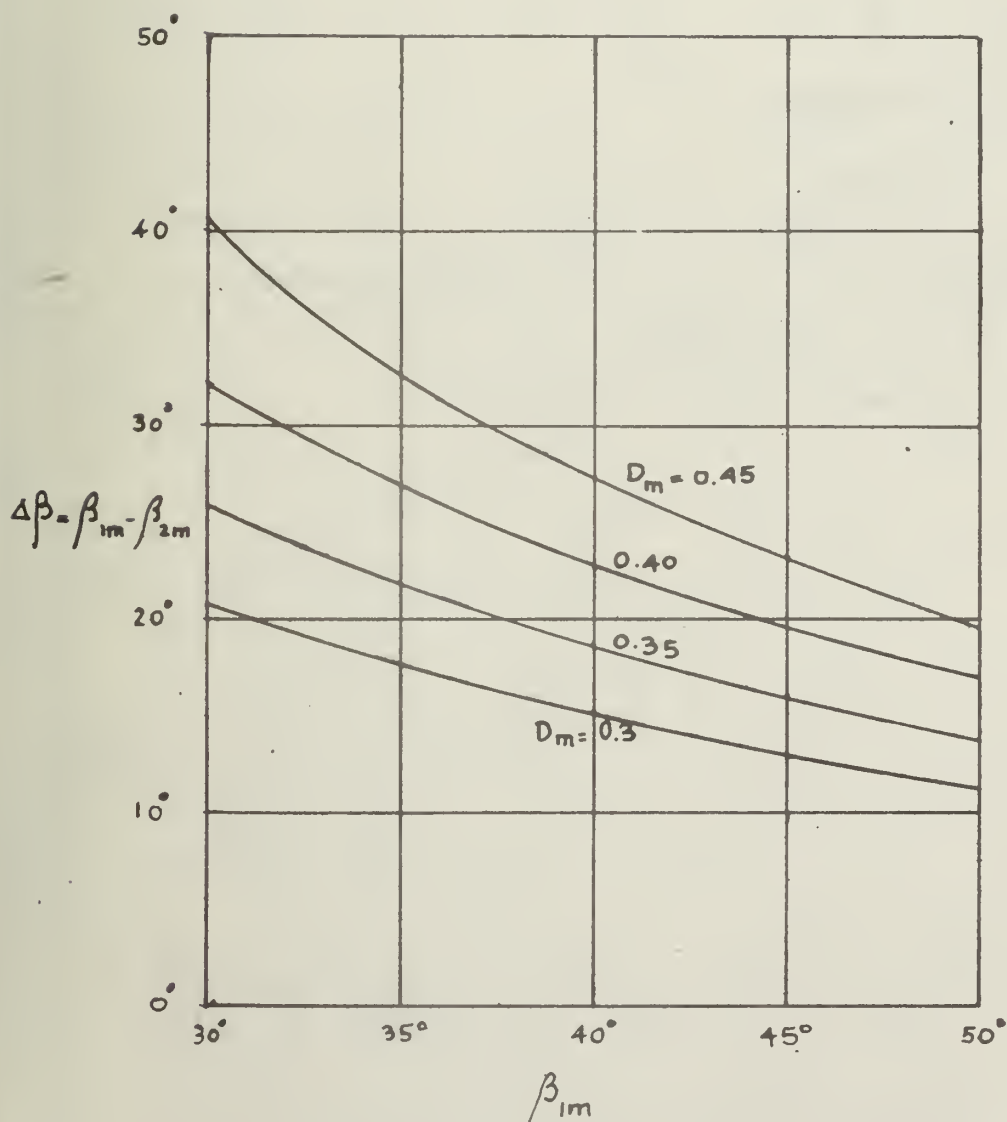


FIG. 8 DISTRIBUTION OF AXIAL VELOCITIES AHEAD AND AFTER ROTOR

V_{a1} = Axial Velocity Component ahead of Rotor

V_{a2} = Axial Velocity Component after Rotor

ωR_m = Peripheral Speed at Arithmetic Mean Stage Radius

———— First Approximation (Table IV) for $\beta_{1a} = 40^\circ, D_m = 0.35$,
for $V_{a1} = V_{a2} = V_m$ at $R/R_m = 1$

----- Second (Final) Approximation (Table VI)

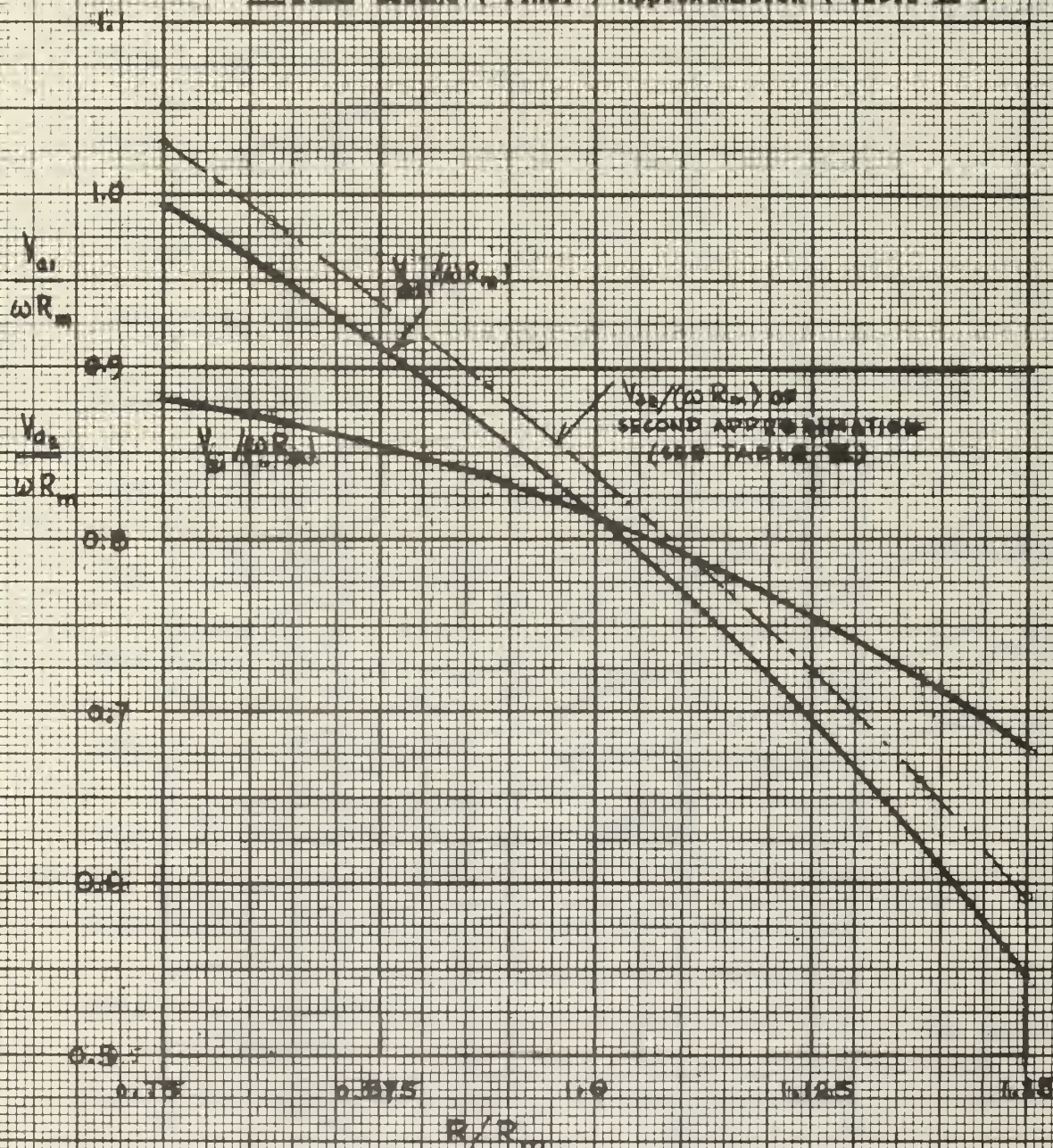


FIG. 8 RELATIVE VOLUME FLOW RATES AHEAD OF AND AFTER ROTOR
OF FIRST APPROXIMATION (SEE TABLE V)

Q_1/Q_{1max} - Percentage of Total Volume Flow Rate ahead of Rotor
between Radius R_1 and Hub Radius

Q_2/Q_{2max} - Percentage of Total Volume Flow Rate after Rotor
between Radius R_2 and Hub Radius

R_m - Arithmetic Mean Stage Radius

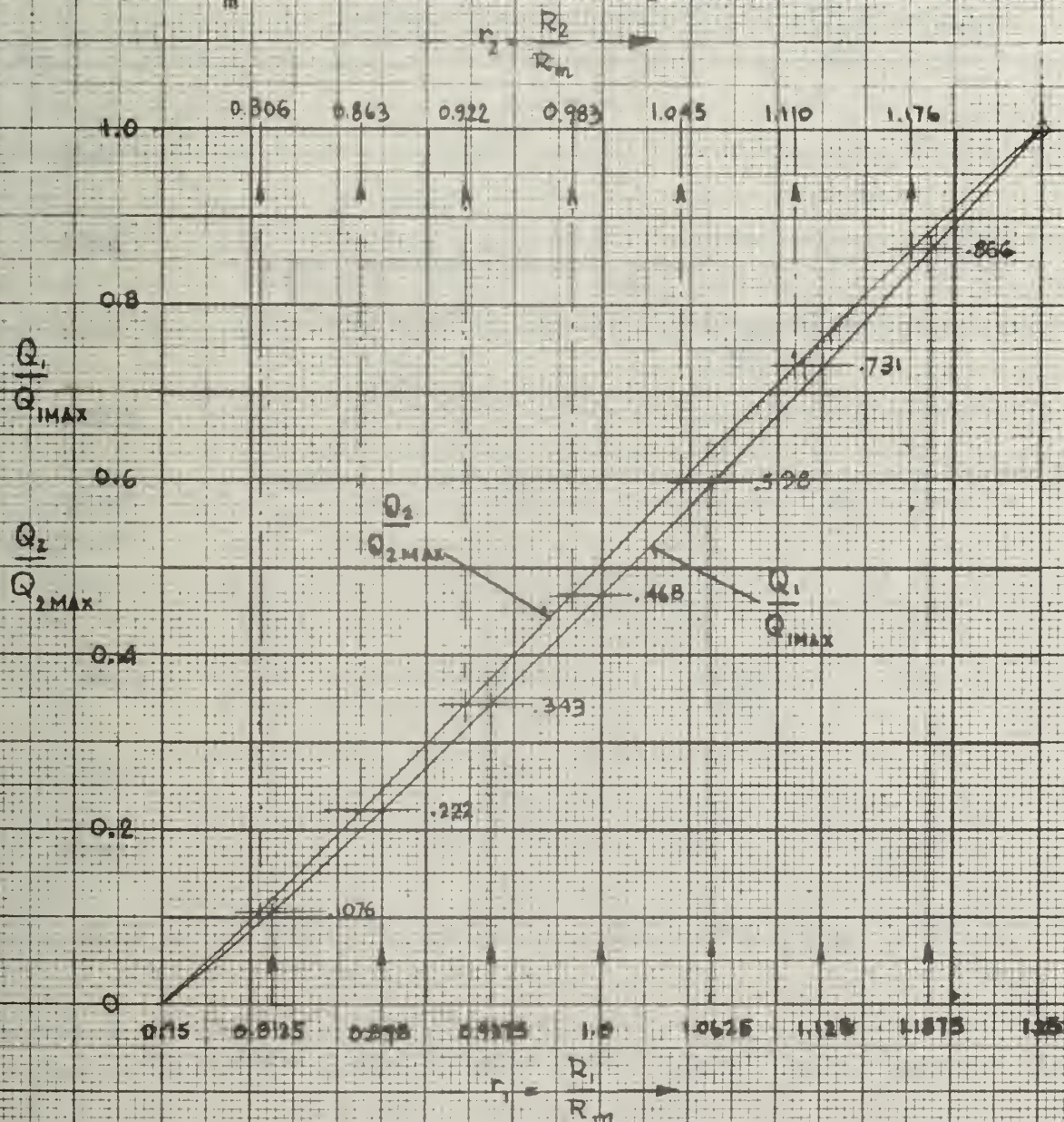


FIG. 10 RADIAL SHIFT OF STREAM SURFACES IN ROTOR FROM DATA OF FIRST APPROXIMATION

R_1 - Radius of Stream Surface ahead of Rotor

R_2 - Radius of Stream Surface after Rotor

R_m - Arithmetic Mean Stage Radius

■ Data Points from Fig. 9

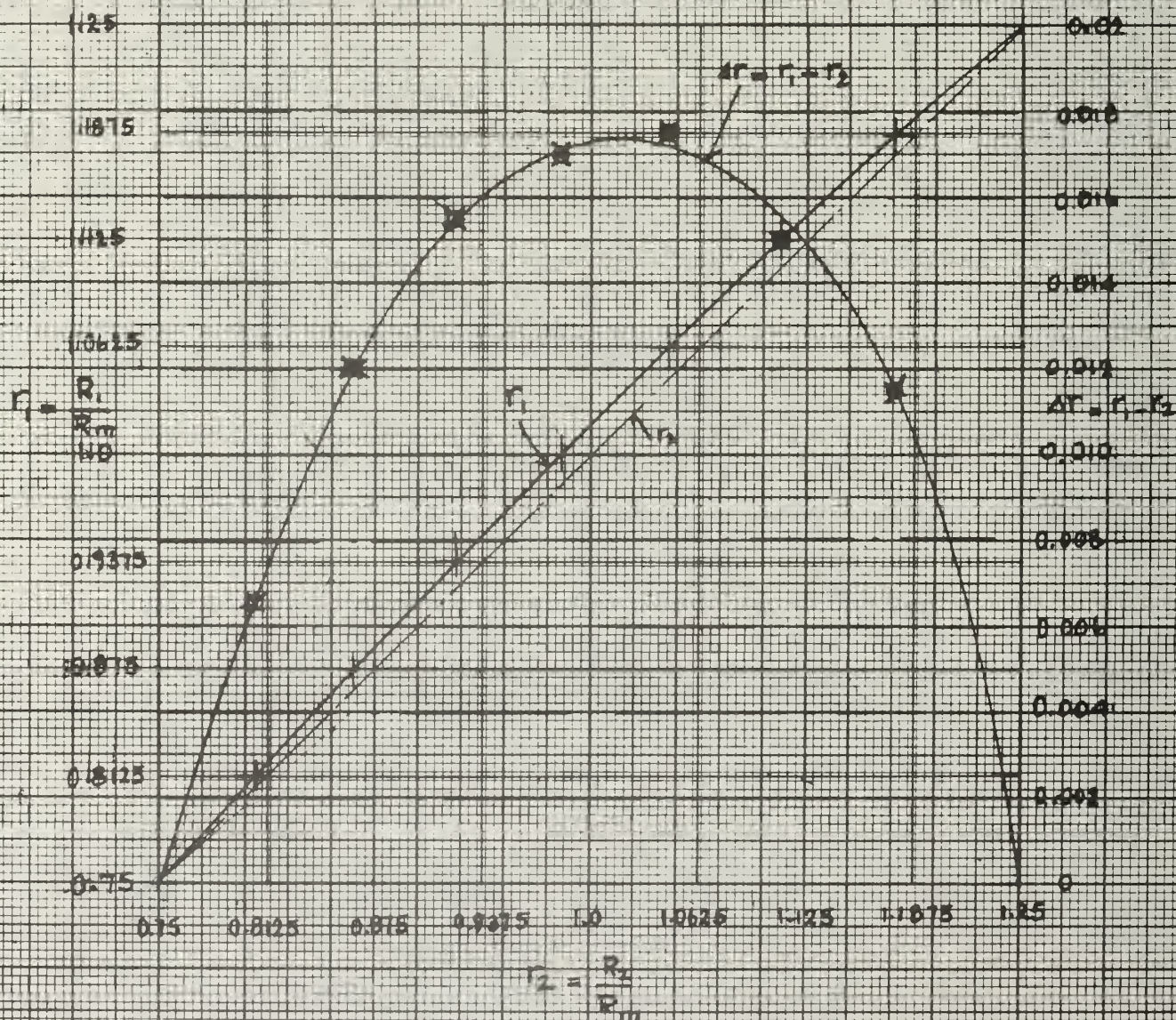


FIG.11 FUNCTION $f(r_2)$ AND $\int_1^{r_2} f(r_2) dr_2$ FOR SECOND APPROXIMATION

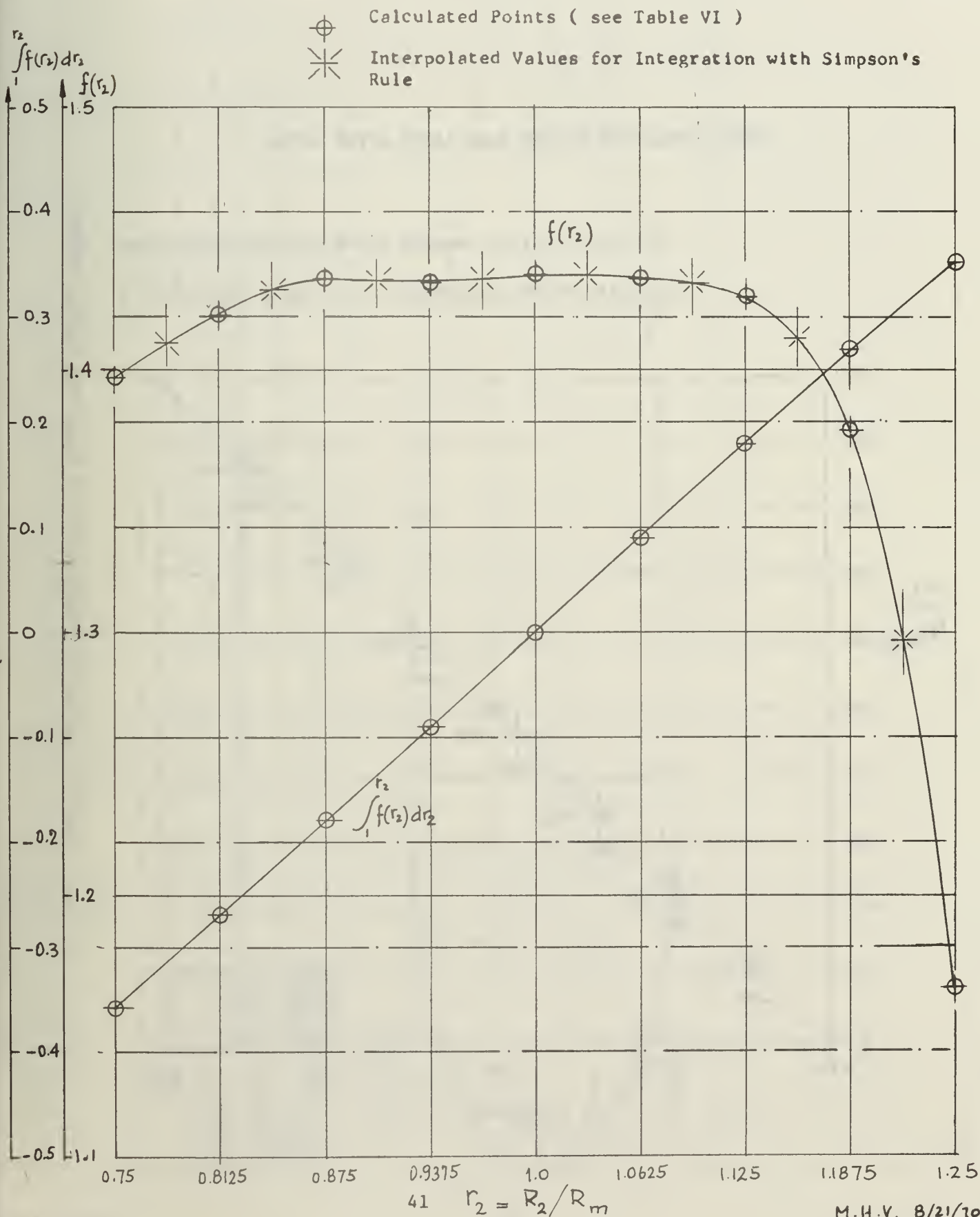
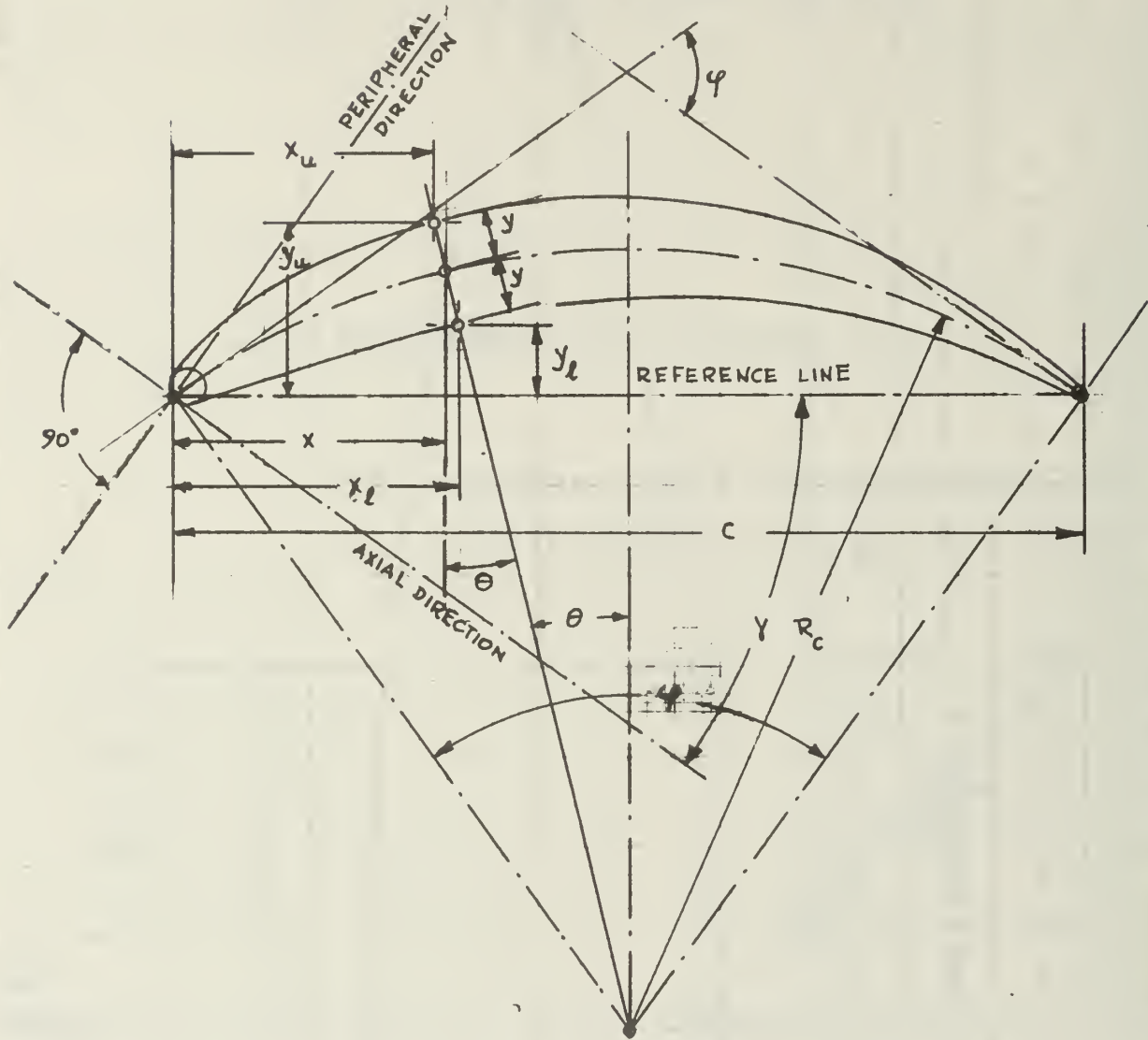


FIG.14 DETERMINATION OF PROFILE COORDINATES



GIVEN: CHORD C ; MAX. THICKNESS t ; CAMBER ANGLE ϕ ; STAGGER ANGLE γ

$$R_c = \frac{C}{2 \sin(\phi/2)} ; \quad \sin \theta = \frac{\frac{C}{2} - x}{R_c} = \left(1 - \frac{2x}{C}\right) \sin(\phi/2) = \left(1 - \frac{2\xi}{100}\right) \sin(\phi/2)$$

$$\frac{x}{C} = \frac{\xi}{100} ; \quad \frac{y}{C} = \frac{\eta}{100} \frac{t}{C}$$

$$\left. \begin{matrix} x_l \\ x_u \end{matrix} \right\} = x \pm y \sin \theta = C \frac{\xi}{100} \pm C \frac{\eta}{100} \frac{t}{C} \left(1 - \frac{2\xi}{100}\right) \sin(\phi/2)$$

$$\left. \begin{matrix} x_l/C \\ x_u/C \end{matrix} \right\} = \frac{\xi}{100} \pm \frac{\eta t}{100 C} \left(1 - \frac{2\xi}{100}\right) \sin(\phi/2)$$

$$\left. \begin{matrix} y_l \\ y_u \end{matrix} \right\} = R_c [\cos \theta - \cos(\phi/2)] \mp y \cos \theta = \frac{C}{2 \sin(\phi/2)} [\cos \theta - \cos(\phi/2)] \mp \frac{\eta}{100} \frac{t}{C} C \cos \theta$$

$$\left. \begin{matrix} y_l/C \\ y_u/C \end{matrix} \right\} = \cos \theta \left[\frac{1}{2 \sin(\phi/2)} \mp \frac{\eta}{100} \frac{t}{C} \right] - \frac{1}{2} \cot(\phi/2)$$

FIG.15 CALCULATED PRESSURE DISTRIBUTIONS IN STAGE (SEE TABLE X)

P_t = Total Pressure , p = Static Pressure

R_t = Radius, ω = Angular Velocity, ρ = Mass Density

Subscripts: (1) Ahead of Rotor

(2) After Rotor, Ahead of Stator

(3) After Stator

(m) Arithmetic Mean Radius of Blading

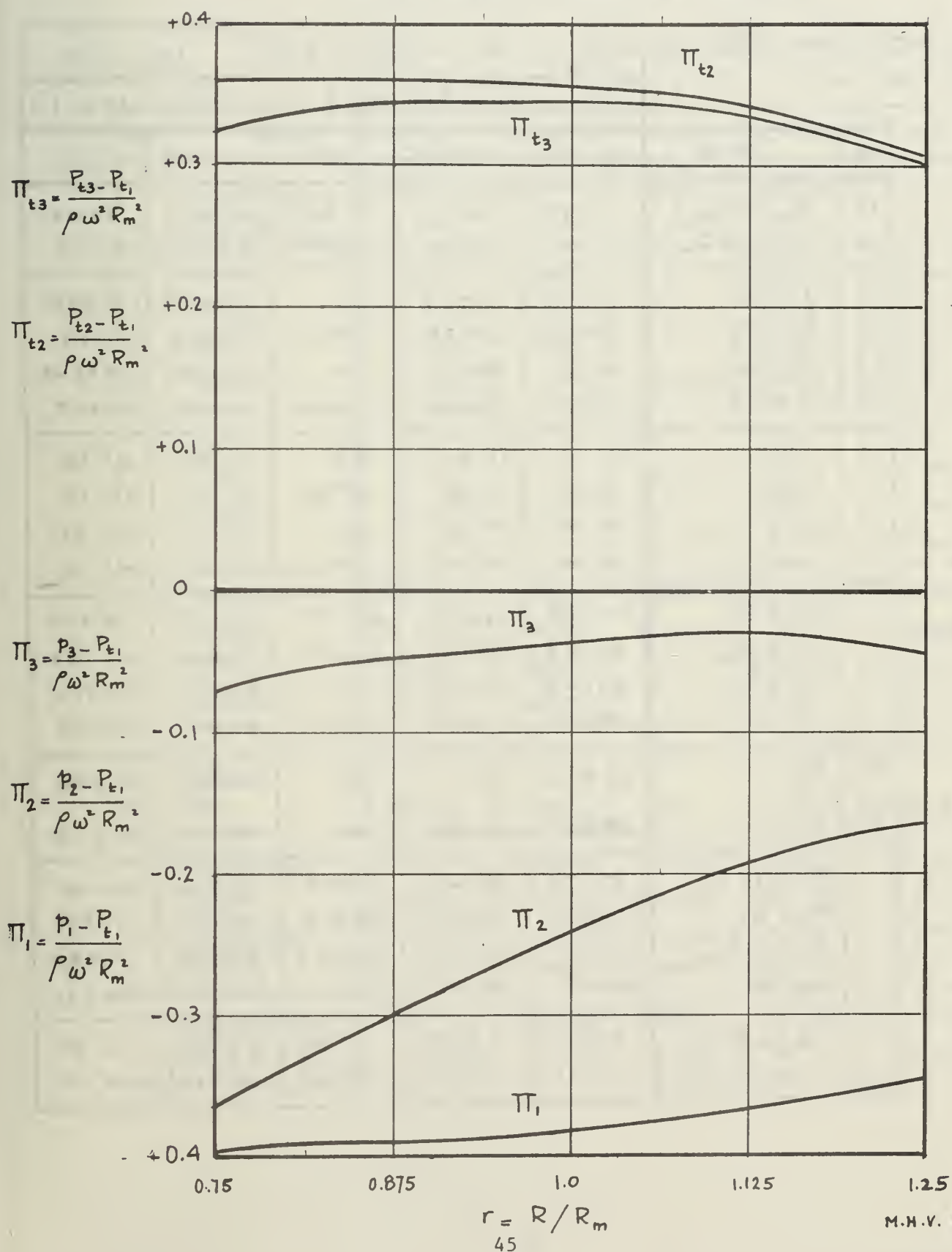


TABLE I

THREE-DIMENSIONAL FLOW CONDITIONS IN COMPRESSOR STAGE WITH VELOCITY DIAGRAM
OF FIG. 2 ON CYLINDRICAL STREAM SURFACES

DATA AT MEAN RADIUS R_m :

		$\beta_{1m} = 45^\circ$		$D_m = 0.40$		$\sigma_m = 1.0$	
		$X = .35206 \text{ (Eq. 4)}$		$Y = .45701 \text{ (Eq. 11)}$			
Eq.	R/R_m	0.75	0.875	1.0	1.125	1.25	
10	$V_{a1}/(\omega R_m)$.75794	.72607	.67602	.60470	.50430	
12	$V_{a2}/(\omega R_m)$.88150	.78818	.67602	.53173	.31177	
13	$\tan \beta_1$.80443	.87964	1.0	1.18897	1.51859	
14	$\tan \beta_2$.15915	.29983	.479231	.76360	1.55299	
16	$\tan \alpha_2$.69167	.81032	1.0	1.35213	2.45638	
15	$\tan \alpha_1$.18509	.32548	.47923	.67223	.96009	
	β_1	$38^\circ 49'$	$41^\circ 20'$	45°	$49^\circ 56'$	$56^\circ 38'$	
	β_2	$9^\circ 03'$	$16^\circ 41'$	$25^\circ 36'$	$37^\circ 22'$	$57^\circ 13'$	
	α_2	$34^\circ 40'$	$39^\circ 01'$	45°	$53^\circ 31'$	$67^\circ 51'$	
	α_1	$10^\circ 29'$	$18^\circ 02'$	$25^\circ 36'$	$33^\circ 55'$	$43^\circ 50'$	
	$\cos \beta_1$.77918	.75085	.70711	.64367	.54997	
	$\cos \beta_2$.98757	.95787	.90179	.79478	.54139	
	$\cos \alpha_2$.82244	.77694	.70711	.59462	.37706	
	$\cos \alpha_1$.98330	.95090	.90179	.82991	.72135	
21	D_R	.26335	.33111	.40	.47523	.56395	
22	D_S	.44507	.42085	.40	.38204	.36738	
	$W_1/(\omega R_m)$.97274	.96700	.95043	.93946	.91696	
	W_2/W_1	.91761	.85093	.78412	.71214	.62802	
	$V_2/(\omega R_m)$	1.07181	1.01447	.95043	.89423	.82684	
	V_1/V_2	.71917	.75267	.78412	.81481	.84551	
	$\Delta \beta = \beta_1 - \beta_2$	$29^\circ 46'$	$24^\circ 39'$	$19^\circ 24'$	$12^\circ 34'$	$-0^\circ 35'$	
	$\Delta \alpha = \alpha_2 - \alpha_1$	$24^\circ 11'$	$20^\circ 59'$	$19^\circ 24'$	$19^\circ 36'$	$24^\circ 01'$	

TABLE II
THREE-DIMENSIONAL FLOW CONDITIONS IN COMPRESSOR STAGE WITH VELOCITY DIAGRAM
OF FIG.2 ON CYLINDRICAL STREAM SURFACES

DATA AT MEAN RADIUS R_m :

		$\beta_{1m} = 45^\circ$		$D_m = 0.35$		$\sigma_m = 1.0$
		$X = .28783 \text{ (Eq.4)}$		$Y = .41462 \text{ (Eq.11)}$		
Eq.	R/R_m	0.75	0.875	1.0	1.125	1.25
10	$V_{a1}/(\omega R_m)$.74200	.70240	.64391	.56188	.44452
12	$V_{a2}/(\omega R_m)$.84627	.75515	.64391	.49790	.26295
13	$\tan \beta_1$.7640	.85703	1.0	1.22878	1.66501
14	$\tan \beta_2$.21638	.36155	.55300	.87282	1.93903
16	$\tan \alpha_2$.66987	.79716	1.0	1.38667	2.81472
15	$\tan \alpha_1$.24678	.38870	.55300	.77343	1.14701
	β_1	37° 23'	40° 36'	45°	50° 51'	59° 01'
	β_2	12° 12'	19° 53'	28° 57'	41° 07'	62° 43'
	α_2	33° 49'	38° 34'	45°	54° 12'	70° 26'
	α_1	13° 52'	21° 15'	28° 57'	37° 43'	48° 46'
	$\cos \beta_1$.79463	.75930	.70711	.63121	.51487
	$\cos \beta_2$.97738	.94042	.87510	.75339	.45836
	$\cos \alpha_2$.83082	.78195	.70711	.58492	.33477
	$\cos \alpha_1$.97087	.93206	.87510	.79101	.65715
21	D_R	.22685	.28753	.35	.41925	.50222
22	D_S	.39098	.36868	.35	.33459	.32203
	$W_1/(\omega R_m)$.93377	.92506	.91062	.89016	.86336
	W_2/W_1	.92727	.86804	.80803	.74242	.66447
	$V_2/(\omega R_m)$	1.01860	.96573	.91062	.85123	.78546
	V_1/V_2	.75031	.78034	.80803	.83448	.86119
	$\Delta\beta = \beta_1 - \beta_2$	25° 11'	20° 43'	16° 03'	9° 44'	- 3° 42'
	$\Delta\alpha = \alpha_2 - \alpha_1$	19° 57'	17° 19'	16° 03'	16° 29'	21° 40'

TABLE III

THREE-DIMENSIONAL FLOW CONDITIONS IN COMPRESSOR STAGE WITH VELOCITY DIAGRAM
OF FIG. 2 ON CYLINDRICAL STREAM SURFACES

DATA AT MEAN RADIUS R_m :		$\beta_{1m} = 40^\circ$	$D_m = 0.40$		$\sigma_m = 1.0$	
		$X = .45911$ (Eq. 4)		$Y = .75594$ (Eq. 11)		
Eq.	R/R_m	0.75	0.875	1.0	1.125	1.25
10	$V_{a1}/(\omega R_m)$.91793	.90101	.86944	.82292	.75969
12	$V_{a2}/(\omega R_m)$	1.05203	.96666	.86944	.75436	.61012
13	$\tan \beta_1$.74196	.77674	.83911	.93150	1.06444
14	$\tan \beta_2$.06552	.18119	.31106	.47517	.72339
16	$\tan \alpha_2$.64737	.72399		1.01616	1.32538
15	$\tan \alpha_1$.87508	.19439		.43558	.58097
	β_1	36°35'	37°50'	40°	42°58'	46°47'
	β_2	3°45'	10°16'	17°16'	25°25'	35°53'
	α_2	32°55'	35°54'		45°28'	52°58'
	α_1	4°18'	11°0'		23°32'	30°09'
	$\cos \beta_1$.80300	.78975	.76604	.73172	.68476
	$\cos \beta_2$.99785	.98398	.95487	.90322	.81023
	$\cos \alpha_2$.83945	.81000		.70142	.60230
	$\cos \alpha_1$.99719	.98165		.91680	.86467
21	D_R	.27845	.34012	.4	.46148	.52821
22	D_S	.44866	.42325	.4	.37884	.35928
	$W_1/(\omega R_m)$	1.14301	1.14088	1.13498	1.12464	1.10952
	W_2/W_1	.92238	.86109	.80224	.74263	.67869
	$V_2/(\omega R_m)$	1.25324	1.19341	1.13498	1.07547	1.01298
	V_1/V_2	.73451	.76910	.80224	.83461	.86733
	$\Delta\beta = \beta_1 - \beta_2$	32°50'	27°34'	22°44'	17°33'	10°54'
	$\Delta\alpha = \alpha_2 - \alpha_1$	28°37'	24°54'	22°44'	21°56'	12°49'

TABLE IV

THREE-DIMENSIONAL FLOW CONDITIONS IN COMPRESSOR STAGE WITH VELOCITY DIAGRAM
OF FIG.2 ON CYLINDRICAL STREAM SURFACES

DATA AT MEAN RADIUS R_m :

		$\beta_{1m} = 40^\circ$		$D_m = 0.35$		$\sigma_m = 1.0$
		$X = .36578 \text{ (Eq. 4)}$		$Y = .66233 \text{ (Eq. 11)}$		
Eq.	R/R_m	0.75	0.875	1.0	1.125	1.25
10	$V_{a1}/(\omega R_m)$.88082	.85479	.81383	.75670	.68022
12	$V_{a2}/(\omega R_m)$.99313	.91014	.81383	.69745	.54722
13	$\tan \beta_1$.70259	.75635	.83911	.95820	1.13391
14	$\tan \beta_2$.13205	.25103	.38965	.57341	.87476
16	$\tan \alpha_2$.62312	.71033	.83910	1.03957	1.40948
15	$\tan \alpha_1$.14889	.23575	.38965	.52850	.70372
	β_1	35° 05'	37° 06'	40°	43° 46'	48° 35'
	β_2	7° 31'	14° 06'	21° 17'	29° 50'	41° 11'
	α_2	31° 56'	35° 23'		46° 07'	54° 39'
	α_1	8° 28'	13° 16'		27° 51'	35° 08'
	$\cos \beta_1$.81824	.79756	.76604	.72204	.66143
	$\cos \beta_2$.99139	.96990	.93176	.86750	.75266
	$\cos \alpha_2$.84871	.81525		.69325	.57863
	$\cos \alpha_1$.98910	.97331		.88412	.81780
21	D_R	.23932	.29511	.35	.40738	.47087
22	D_S	.39528	.37717	.35	.33105	.31387
	$W_1/(\omega R_m)$	1.07649	1.07175	1.06238	1.04801	1.02841
	W_2/W_1	.93056	.87552	.82214	.76712	.70695
	$V_2/(\omega R_m)$	1.17016	1.11639	1.06238	1.00605	.94571
	V_1/V_2	.76101	.78665	.82214	.85072	.87950
	$\Delta \beta = \beta_1 - \beta_2$	27° 34'	23°	18° 43'	13° 56'	7° 24'
	$\Delta \alpha = \alpha_2 - \alpha_1$	23° 28'	22° 07'	18° 43'	18° 16'	19° 31'

TABLE V FLOW RATE CALCULATIONS FROM FIRST APPROXIMATION

EQ	$r_1 = R_1/R_m, r_2 = R_2/R_m$	0.75	0.8125	0.875	0.9375	1.0	1.0625	1.125	1.1875	1.25
10	$V_{a1}^* = V_{a1}/(\omega R_m)$.88082	.86965	.85479	.83622	.81383	.78743	.75670	.72119	.68022
12	$V_{a2}^* = V_{a2}/(\omega R_m)$.99313	.95300	.91014	.86399	.81383	.75875	.69745	.62801	.54722
49	Q_1^*	0	.042725	.088179	.136051	.185982	.237559	.290307	.343673	.397007
	Q_{1max}^*									.397007
	$Q_1^*/Q_{1max}^* = Q_1/Q_{1max}$	0	.107618	.222109	.342692	.468460	.598375	.731239	.865660	1
52	Q_2^*	0	.047474	.096558	.146757	.197501	.248126	.297839	.345664	.390345
	Q_{2max}^*									.390345
	$Q_2^*/Q_{2max}^* = Q_2/Q_{2max}$	0	.121621	.247366	.375967	.505965	.635658	.763015	.885535	1

Eq.	$r_2 = R_2/R_m$	0.75	0.8125	0.875	0.9375	1.0	1.0625	1.125	1.1875	1.25
55	r_1	.75	.8198	.8879	.9535	1.0173	1.0794	1.1399	1.1979	1.25
46	$\partial r_1 / \partial r_2$	1.129	1.102	1.072	1.036	1.008	.980	0.952	.900	.755
47	$f(r_2)$	1.39736	1.42030	1.43482	1.43293	1.43629	1.43447	1.42774	1.37672	1.16467
	$\int_1^{r_2} f(r_2) dr_2$	-.35669	-.26859	-.17926	-.08964	0	.08972	.17923	.26726	.35194
47	$V_{m2}^* = 0.844$	1.03404	.99042	.94424	.89553	<u>0.844</u>	.78906	.73014	.66714	.60033
52	Q_2^*									<u>.408091</u> *
47	$V_{m2}^* = 0.8382$	1.02921	.98548	.93906	.89006	<u>0.8382</u>	.78285	.72343	.65978	.59215
52	Q_2^*									<u>.405012</u> *
47	$V_{m2}^* = 0.838$	1.02904	.98531	.93888	.88988	<u>0.838</u>	.78264	.72320	.65953	.59187
52	Q_2^*	0	.049136	.099826	.151569	.203827	.256001	.307412	.357312	.404907
	Q_2/Q_{2max}	0	.121350	.246542	.374332	.503390	.63225	.75922	.88246	1
47	$V_{m2}^* = 0.837$	1.02823	.98446	.93799	.88893	<u>0.837</u>	.78157	.72204	.65826	.59045
52	Q_2^*									<u>.404375</u> *

* Required Value for $Q_{2max}^* = 0.404347$ to satisfy Continuity with blockage Factor $k_B = 0.98$

At $V_{m2}^* = 0.837$ the Value of Q_{2max}^* is 99.99% of Required Value

TABLE VII SUMMARY OF FLOW PROPERTIES IN STAGE

Eq.	TABLE	$r_1 = R_1/R_m$	0.75	0.8125	0.875	0.9375	1.0	1.0625	1.125	1.1875	1.25
56	V	$V_{a1}^* = V_{a1}/(\omega R_m)$.88082	.86965	.85479	.83622	.81383	.78743	.75670	.72119	.68022
57		V_{u1}^*	.131147	.181155	.228483	.273667	.317110	.359118	.399931	.439737	.478688
58		W_{u1}^*	.618853	.631345	.646517	.663833	.682890	.703382	.725069	.747763	.771312
59		$\tan \alpha_1$.148892	.208302	.267297	.327267	.389651	.456064	.528520	.609739	.703725
63		$\tan \beta_1$.702588	.725976	.756346	.793849	.839106	.893262	.958198	1.036845	1.133915
64		$\cos \alpha_1$.989097	.978988	.966083	.950399	.931764	.909845	.884114	.853803	.817800
		$\cos \beta_1$.818236	.809235	.797564	.783213	.766042	.745787	.722037	.694202	.661430
		V_1^*	.890530	.888318	.884800	.879862	.873429	.865465	.855885	.844680	.831770
		W_1^*	1.076487	1.074657	1.071751	1.067679	1.062383	1.055837	1.048007	1.038876	1.028407
Eq.	TABLE	$r_2 = R_2/R_m$	0.75	0.8125	0.875	0.9375	1.0	1.0625	1.125	1.1875	1.25
45	VI VII	r_1 for same stream surface	.75	.8198	.8879	.9535	1.0173	1.0794	1.1399	1.1979	1.25
60		$V_{a2}^* = V_{a2}/(\omega R_m)$	1.02904	.98531	.93888	.88988	.838	.78264	.72320	.65953	.59187
61		V_{u2}^*	.618853	.638678	.659512	.679969	.700340	.720416	.740067	.758208	.771312
62		W_{u2}^*	.131147	.173822	.215488	.257531	.299660	.342084	.384933	.429292	.478688
		$\tan \alpha_2$.601389	.648200	.702446	.764113	.835727	.920495	1.023323	1.149619	1.303178
		$\tan \beta_2$.127446	.176413	.229516	.289400	.357590	.437090	.532263	.650906	.808772
65		$\cos \alpha_2$.856967	.839133	.818291	.794585	.767317	.735750	.698909	.656302	.608775
66		$\cos \beta_2$.991976	.984793	.974658	.960583	.941608	.916295	.882745	.838096	.777530
		V_2^*	1.200793	1.174200	1.147367	1.119931	1.092117	1.063731	1.034755	1.004918	.972231
		W_2^*	1.037363	1.000525	.963292	.926395	.889966	.854135	.819263	.786938	.761218
		$\Delta V_u^* = \Delta W_u^* = V_{u2}^* - V_{u1}^*$.487707	.457523	.431029	.406302	.383230	.361298	.340136	.318471	.292624
17	ROTOR	W_2/W_1	.963656	.931018	.898802	.867672	.837708	.808965	.781734	.757490	.740191
68		$(D_2^*)_0 (D_{EQR})_0$.2062	.2419	.2771	.3107	.3426	.3728	.4008	.4245	.4376
18	STATOR	V_1/V_2	1.3284	1.3749	1.4273	1.4811	1.5352	1.5896	1.6431	1.6912	1.7205
69		$(D_2^*)_0 (D_{EQR})_0$.74162	.75653	.77157	.78564	.79976	.81360	.82714	.84055	.85553
			.4107	.4018	.3932	.3844	.3757	.3668	.3578	.3476	.3326
			1.7416	1.708	1.6749	1.6419	1.6088	1.5750	1.5043	1.5037	1.4611

TABLE VIII ROTOR BLADE PROFILES FOR ASSUMED CHORDS AND THICKNESSES
NUMBER OF ROTOR BLADES = 30

EQ.	TABLE	REF. 6 FIG. PAGE	$r = R/R_m$	0.75	0.8125	0.875	0.9375	1.0	1.0625	1.125	1.1875	1.25
			R (in.)	10.8	11.7	12.6	13.5	14.4	15.3	16.2	17.1	18.0
			ASSUMED CHORD C_R (in.)	2.4	2.5	2.6	2.7	2.8	2.9	3.0	3.1	3.2
			SOLIDITY σ_R	1.061	1.020	.985	.955	.928	.905	.884	.865	.849
17			D_R	.2498	.2777	.3053	.3316	.3566	.3801	.4018	.4197	.4274
68			D_{EQR}	1.3662	1.4105	1.4567	1.5034	1.5506	1.5978	1.6442	1.6855	1.7085
			ASSUMED MAX. THICK. (in.)	.30	.276	.256	.239	.225	.214	.206	.201	.20
			t/C_R THICKN. / CHORD	.125	.1104	.0985	.085	.0764	.0738	.0687	.0648	.0625
	VII		β_1 (°)	35.09	35.99	37.47	38.44	40.0	41.77	43.78	46.04	48.59
	VII		β_2 (°)	7.26	10.0	12.93	16.14	19.68	23.61	28.02	33.06	38.96
			$\Delta\beta = \beta_1 - \beta_2$ (°)	27.83	25.99	24.54	22.30	20.32	18.16	15.76	12.98	9.63
		137 195	$(i_o)_{10}$ (°)	3.00	2.95	2.90	2.90	2.95	3.00	3.05	3.10	3.20
		142 199	$(K_i)_t$ (°)	1.10	1.045	.995	.930	.890	.865	.832	.810	.790
		161 212	$(\delta_o)_{10}$ (°)	.73	.73	.77	.79	.82	.84	.91	.97	1.06
		172 219	$(K_s)_t$ (°)	1.40	1.145	.975	.80	.70	.67	.615	.575	.55
		168 217	m	.263	.272	.281	.29	.30	.308	.318	.328	.335
		138 196	n	-.11	-.117	-.126	-.135	-.145	-.155	-.166	-.180	-.197
71			i_o (°)	3.63	3.39	3.17	2.97	2.89	2.85	2.79	2.76	2.78
72			δ_o (°)	1.12	.92	.82	.69	.63	.62	.62	.61	.64
70			φ (°)	40.383	38.494	37.4198	34.817	32.540	29.665	26.337	22.012	16.004
70			φ (°, ')	40° 23'	38° 30'	37° 25'	34° 49'	32° 32'	29° 40'	26° 20'	22° 01'	16° 0'
73			l (°)	-.812	-1.114	-1.545	-1.730	-1.828	-1.748	-1.582	-1.202	-.373
74			δ (°)	11.74	11.39	11.33	10.79	10.39	9.76	8.99	7.83	6.0
75			γ (°)	15.711	17.857	20.305	22.761	25.558	28.686	32.193	36.236	40.96
75			γ (°, ')	15° 43'	17° 51'	20° 18'	22° 46'	25° 33'	28° 41'	32° 12'	36° 14'	40° 58'
			$l_{axR} = C_R \cos \gamma$ (in.)	2.310	2.380	2.438	2.490	2.526	2.544	2.538	2.500	2.416

TABLE IX STATOR BLADE PROFILES FOR ASSUMED CHORDS AND THICKNESSES

NUMBER OF STATOR BLADES = 32

EQ. TABLE	REF. FIG.	PAGE	$r = R/R_m$	0.75	0.8125	0.875	0.9375	1.0	1.0625	1.125	1.1875	1.25
			R (in.)	10.8	11.7	12.6	13.5	14.4	15.3	16.2	17.1	18.0
			ASSUMED CHORD c_s (in.)	2.60	2.55	2.50	2.45	2.40	2.35	2.30	2.25	2.20
			SOLIDITY σ_B	1.226	1.110	1.010	.924	.849	.782	.723	.670	.622
18			D_S	.4240	.4190	.4147	.4106	.4073	.4035	.4001	.3959	.3762
69			D_{eqs}	1.7612	1.7328	1.7049	1.6752	1.6459	1.5774	1.5772	1.5477	1.5044
			ASSUMED MAX. THICKNESS (in.)	.170	.180	.190	.200	.210	.220	.230	.240	.250
			t/c_s THICKN./CHORD	.065	.070	.076	.082	.087	.094	.100	.107	.114
VII			α_2 (°)	31.039	32.968	35.086	37.367	39.886	42.629	45.660	48.981	52.449
VII			α_1 (°)	8.468	11.767	14.965	18.122	21.288	24.516	27.857	31.372	35.135
			$\Delta\alpha = \alpha_2 - \alpha_1$ (°)	22.571	21.201	20.121	19.245	18.598	18.113	17.803	17.609	17.364
	137	195	$(i_o)_{10}$ (°)	3.00	2.90	2.80	2.75	2.70	2.64	2.60	2.55	2.53
	142	199	$(K_i)_t$.810	.845	.880	.915	.940	.975	1.0	1.030	1.055
	161	212	$(\delta_o)_{10}$ (°)	.65	.67	.70	.75	.77	.81	.83	.87	.90
	142	199	$(K_s)_t$.575	.630	.695	.715	.825	.915	1.0	1.095	1.20
	168	217	m	.226	.250	.272	.300	.321	.345	.375	.405	.431
	138	196	n	-.080	-.099	-.115	-.133	-.154	-.178	-.202	-.231	-.263
71			i_o (°)	2.673	2.695	2.710	2.768	2.792	2.831	2.860	2.889	2.936
72			δ_o (°)	.411	.464	.535	.590	.699	.815	.913	1.048	1.188
70			φ (°)	29.264	29.138	29.276	30.100	31.438	33.746	37.485	43.319	51.033
			φ (°, ')	29°16'	29°08'	29°16'	30°06'	31°26'	33°45'	37°29'	43°19'	51°02'
73			i (°)	.332	-.190	-.657	-1.235	-2.050	-3.176	-4.712	-7.118	-10.485
74			δ (°)	7.024	7.749	8.498	9.620	10.790	12.460	14.970	18.600	23.180
75			γ (°)	16.075	18.588	21.105	23.552	26.216	28.932	31.629	34.439	37.468
			γ (°, ')	16°04'	18°35'	21°06'	23°33'	26°13'	28°56'	31°38'	34°26'	37°28'
			i' (ASSUMED INCIDENCE)						-3.0°	-4.0°	-5.0°	-6.0°
	177	221	$d\delta/di$.122	.155	.190	.232
78			$\varphi = \varphi'$ (°)						33.594	36.908	41.611	47.584
			$\varphi = \varphi'$ (°, ')						33°36'	36°54'	41°37'	47°35'
79			$\gamma = \gamma'$ (°)						28.832	31.206	33.175	34.707
			$\gamma = \gamma'$ (°, ')						28°50'	31°12'	33°13'	34°42'
			$l_{ax's} = c_s \cos \gamma$ (in.)	2.498	2.417	2.332	2.245	2.153	2.059	1.969	1.882	1.809

TABLE X ROTOR AND STATOR LOSSES, PRESSURES AND STAGE EFFICIENCIES

EQ.	TABLE	$r = R/R_m$	0.75	0.8125	0.875	0.9375	1.0	1.0625	1.125	1.1875	1.25
86 85 85	VIII	σ_R	1.061	1.020	.985	.955	.928	.905	.884	.865	.849
	VIII	$D_R \cos \beta_2$.2498	.2777	.3053	.3316	.3566	.3801	.4018	.4197	.4274
	VII	$1 - \lambda_R$ $Y_R \cos \beta_2 / (2\sigma_R)$ Y_R	.99198 0 .00527 .01128	.98479 .125 .00575 .0190	.97466 .25 .00634 .01141	.96058 .375 .00742 .01475	.94161 .5 .00951 .01875	.91629 .625 .01362 .02690	.88274 .75 .02097 .04200	.83810 .875 .0331 .06876	.77753 1.0 .05082 .11099
89 88 88	IX	σ_S	1.226	1.110	1.010	.924	.849	.782	.723	.670	.622
	IX	$D_S \cos \alpha_1$.4240	.4190	.4147	.4106	.4073	.4035	.4001	.3959	.3762
	VII	$1 - \lambda_S$ $Y_S \cos \alpha_1 / (2\sigma_S)$ Y_S	.98910 1.0 .01973 .04890	.97899 .875 .01536 .03484	.96608 .75 .01256 .02627	.95040 .625 .01076 .02092	.93176 .50 .00973 .01773	.90984 .375 .00914 .01571	.88411 .25 .00885 .01447	.85380 .125 .00866 .01359	.81780 0 .00801 .01218
91 93 98 99 100	VII	$W_1^* V_1^*$	1.07649	1.07466	1.07175	1.06768	1.06238	1.05584	1.04801	1.03888	1.02841
	VII	V_2^*	.89053	.88832	.88480	.87986	.87343	.86545	.85588	.84468	.83177
	VII		1.20079	1.17420	1.14737	1.11993	1.09218	1.06373	1.03475	1.00492	.97223
		π_{t2}	.35924	.35891	.35923	.35737	.35520	.35079	.34271	.32867	.30709
		$\pi_{t3} \pi_1$.32399	.33489	.34193	.34425	.34463	.34190	.33497	.32181	.30133
94 96		$\pi_2 \pi_3$	-.39652	-.39000	-.39144	-.38708	-.38144	-.37451	-.36627	-.35674	-.34592
			-.36171	-.33046	-.29900	-.26975	-.24116	-.21498	-.19264	-.17625	-.16553
			-.07253	-.05514	-.04950	-.04282	-.03681	-.03261	-.03130	-.03493	-.04459
		η_t	.8857	.9155	.9348	.9411	.9422	.9347	.9158	.8798	.8238
	VII	V_{a1}^*	.88082	.86965	.85479	.83622	.81383	.78743	.75670	.72119	.68022
96		$r, V_{a1}^* \eta_t$.58514	.64692	.69918	.73782	.76676	.78202	.77958	.75347	.70046

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